States, Metastates, and Replica Symmetry Breaking

or

"Is there there a static calculation for spin glasses which describes (non-equilibrium) dynamics?"

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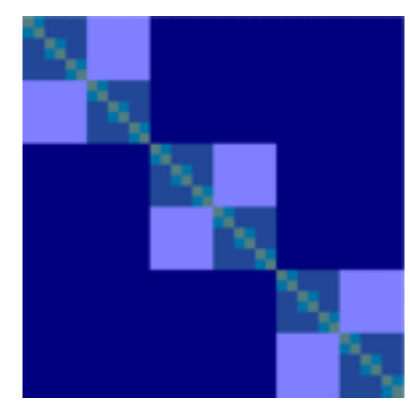


Work in collaboration with Matt Wittmann, Phys. Rev. E 90, 062137 (2014)

Time averages and Gibbs averages

Experiments: time average (dynamics)
Theory: static average (Gibbs) is simpler

Do they agree?
According to ergodic hypothesis: Yes.

But there are exceptions, e.g.
(i) System with a phase transition (e.g. Ising ferromagnet) below T_c.
Experiment: system is **either** in "up" spin state **or** "down" spin state (related by symmetry).
These are "pure states" which have a convenient clustering property:

$$\langle S_i S_j \rangle \to \langle S_i \rangle \langle S_j \rangle \quad \text{for } |R_i - R_j| \to \infty$$

Theory: Gibbs average includes **both** "up" **and** "down" states.
Correlations in the Gibbs state do **not** have the clustering property since $\langle S_i \rangle = 0$ so it is a "mixed state".

(ii) Spin glasses (more complicated).

Pure and mixed states in spin glasses

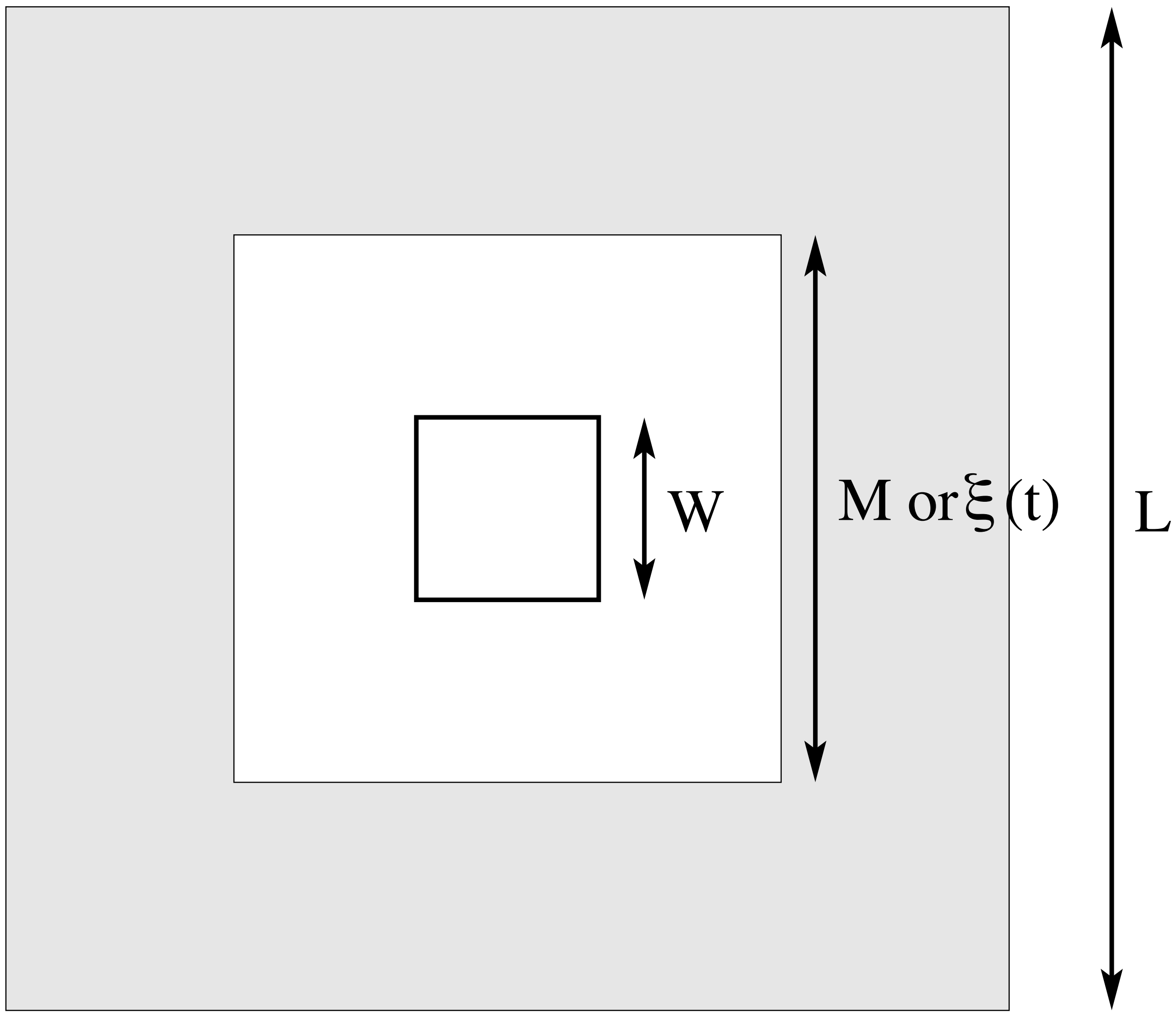
Clustering property is convenient. How can we describe spin glasses in terms of pure states? Extensively discussed by Newman and Stein (NS).

Note: a state (pure or mixed) is defined by a set of correlation functions in a region. Possibility of different pure states, **not related by symmetry**.

Pure states are only rigorously defined in the thermodynamic limit. What limiting procedure can one define as the size increases to infinity? Problem: chaotic dependence of state in central region as one increases the size of the system (NS).

Need to average over the distribution of these states as the system grows. NS (1997) call this the “metastate”.
An alternative definition of the metastate, due to Aizenman and Wehr (AW) (1990), is more convenient for our purposes (Read, 2014). The NS and AW metastates are argued to be equivalent.

The AW Metastate



System is of size L with, say, periodic boundary conditions. Measure correlations in a window of size W. There is also an intermediate scale M where

W << M << L

Outer region (shaded) (outside M)
Inner region (inside M).

Eventually all scales → ∞

- Determine Gibbs average for a given set of bonds.
- Determine the state of the window (pure or mixed) from the correlations in the window.
- Generate the metastate by repeating for many choices of bonds in the outer region.
- Finally average over the bonds in the inner region (not necessary for W → ∞).

Metastate averaged state (MAS) $C_{ij}^{MAS} = [\,\langle S_i S_j \rangle\,]^2_{\mathrm{outer}}$ for i, j in window.
(note location of square)

Different scenarios for the AW Metastate

- For each set of bonds in outer region the window is in a pure state, which is the same for each set of bonds in the outer region. This is the "droplet model" of Fisher and Huse (FH) (1987).
- For each set of bonds in outer region the window is in a pure state, but this state varies chaotically when varying the bonds in the outer region. This is the "chaotic pairs" picture of NS (1992).
- For each set of bonds in the outer region the window is in a mixed state. This mixed state varies chaotically when varying the bonds in the outer region. This is the "replica symmetry breaking (RSB)" picture (Parisi 1979).

Note:

- Since we work in zero field, "pure state" really means a pair of pure states related by time-reversal.
- In the droplet model the state of the system in a local region is independent of all the bonds far away. In the chaotic pairs and RSB the state of the system depends on the set of all far-away bonds (which makes the calculation of the ground state much harder).

MAS correlations for the different scenarios

Recall, the MAS correlation is defined by $C_{ij}^{\mathrm{MAS}} = [\,\langle S_i S_j \rangle\,]^2_{\mathrm{outer}}$

What are the predictions of different scenarios?

- "Droplet model" (FH)

$$\lim_{R_{ij}\to\infty} C_{ij}^{\mathrm{MAS}} = q^2$$

(because of clustering), where q is the spin glass order parameter

- "Chaotic pairs" (NS)

$$C_{ij}^{\mathrm{MAS}} \propto r_{ij}^{-(d-\zeta)}$$

(Read, 2014) for some value of ζ.

- "Replica symmetry breaking (RSB)" (Parisi)

Same power-law decay as for chaotic pairs, with ζ=4 in mean field (d > 6) (Read (2014), Marinari et al (2000), de Dominicis and Kondor).
Except for droplet model, MAS correlations tend to zero at large distance. MAS correlations computed in d=3 by Billoire et al. (2017). Find ζ < d (⇒RSB or chaotic pairs)

(Non-equilibrium) dynamic correlations

Now suppose we quench the system below T_c and observe the time dependence of the correlations. Correlations will develop up to a scale ξ(t) which increases only slowly with t, $\xi(t) \propto t^{1/z(T)}$
How does the non-equilibrium correlation function

$$C_{ij}(t) = [\,\langle S_i(t) S_j(t)\rangle^2\,]$$

vary? Here [...] is an average over **all** bonds. At distances less than ξ(t) a power-law dependence on distance is found so we postulate

$$C_{ij}(t) = \frac{1}{r_{ij}^{\alpha_d}} f\left(\frac{r_{ij}}{\xi(t)}\right)$$

For $r_{ij} << \xi(t)$, we have

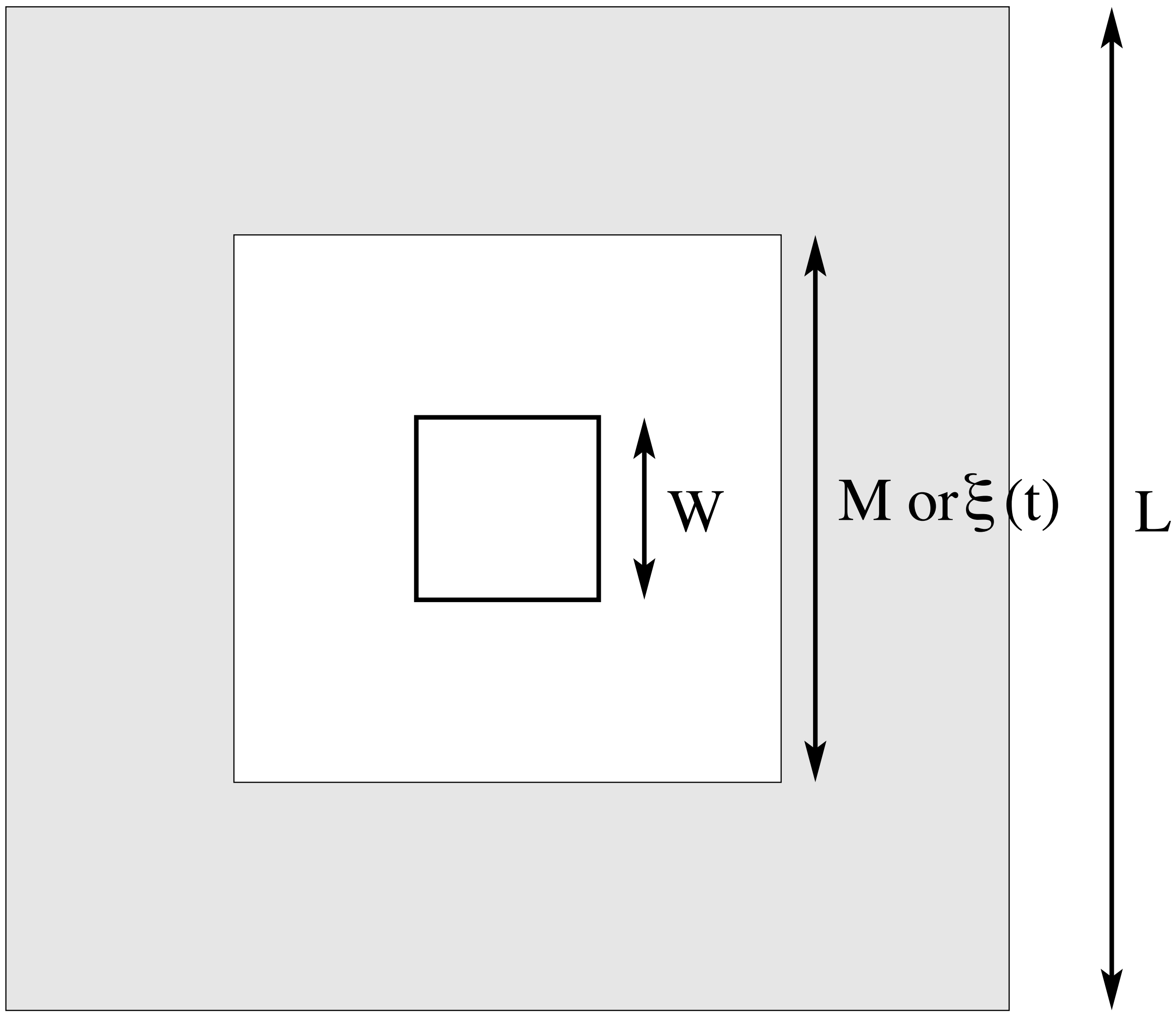
$$C_{ij}(t) \propto \frac{1}{r_{ij}^{\alpha_d}} \qquad (r_{ij} \ll \xi(t))$$

i.e. a power law. For MAS averages we also have a power law:

$$C_{ij}^{\mathrm{MAS}} \propto \frac{1}{r_{ij}^{\alpha_s}}$$

where $\alpha_s = d - 4$ in MF $(d > 6)$.

Connection(?) between MAS average and dynamics



The similarity between MAS averages and non-equilibrium dynamic averages (dynamic metastate) was first pointed out by Fisher and White (2006) who used the term "maturation metastate" to describe the ensemble of states generated dynamically on scales less than $\xi(t)$.

The effects of averaging over the exterior bonds in the MAS average may be analogous to averaging over the (non-equilibrated) spins at distances $> \xi(t)$ in the time-dependent average.

Thus $\xi(t)$ seems to be analogous to the intermediate scale M.

$$C_{ij}^{\mathrm{MAS}} \propto \frac{1}{r_{ij}^{\alpha_s}} \qquad C_{ij}(t) \propto \frac{1}{r_{ij}^{\alpha_d}} \qquad (r_{ij} \ll \xi(t))$$

Is $\alpha_s = \alpha_d$? **A connection between statics and dynamics would be very useful.**

Is $\alpha_s = \alpha_d$?

In d=3, Alvaros Baños et al (2010) find numerically that α_d is close to a quantity similar to α_s.

Here we focus on the mean field limit because there is an **analytic** value for α_s for $d > d_u = 6$, namely $\alpha_s = d - 4$ (Read)

Hence we want to determine α_d for $d > 6$. Difficult because the number of spins N increases too fast with L, $N = L^d$.

Instead we investigate a model in d = 1 with long-range (LR) power-law interactions which, it is argued, is a proxy for a short-rage (SR) model.

Varying the power in the LR model varies the dimension in the corresponding SR model.

The model

The 1d Hamiltonian is given by

$$\mathcal{H} = -\sum_{i<j} J_{ij} S_i S_j \qquad (S_i = \pm 1)$$

where the bonds are independent random variable with mean and variance given by

$$[J_{ij}] = 0, \qquad [J_{ij}^2] \propto \frac{1}{r_{ij}^{2\sigma}}$$

Rather than every spin interacting with every other spin (inefficient for large sizes) we follow Leuzzi et al (2008) and consider a dilute LR model. The probability that a bond exists between i and j is proportional to $1/r_{ij}^{2\sigma}$, but, if present, has variance unity. We take a mean coordination, $z_b = 6$.

We will need large sizes up to $N = 2^{26}$.
Non-dilute model: of order $N^2 = 2^{52} = 4.5 \times 10^{15}$ operations per sweep (huge).
Dilute model: of order $z_b N = 4 \times 10^8$ operations per sweep (manageable).

1000 samples per for each size.

The Parameters

For d > 6 the relation between d and σ is $d = \frac{2}{2\sigma - 1}$

We choose d = 8 so σ = 5/8.
Exponent for SR model in d-dimension is d times exponent for corresponding LR model (Baños et al 2012).

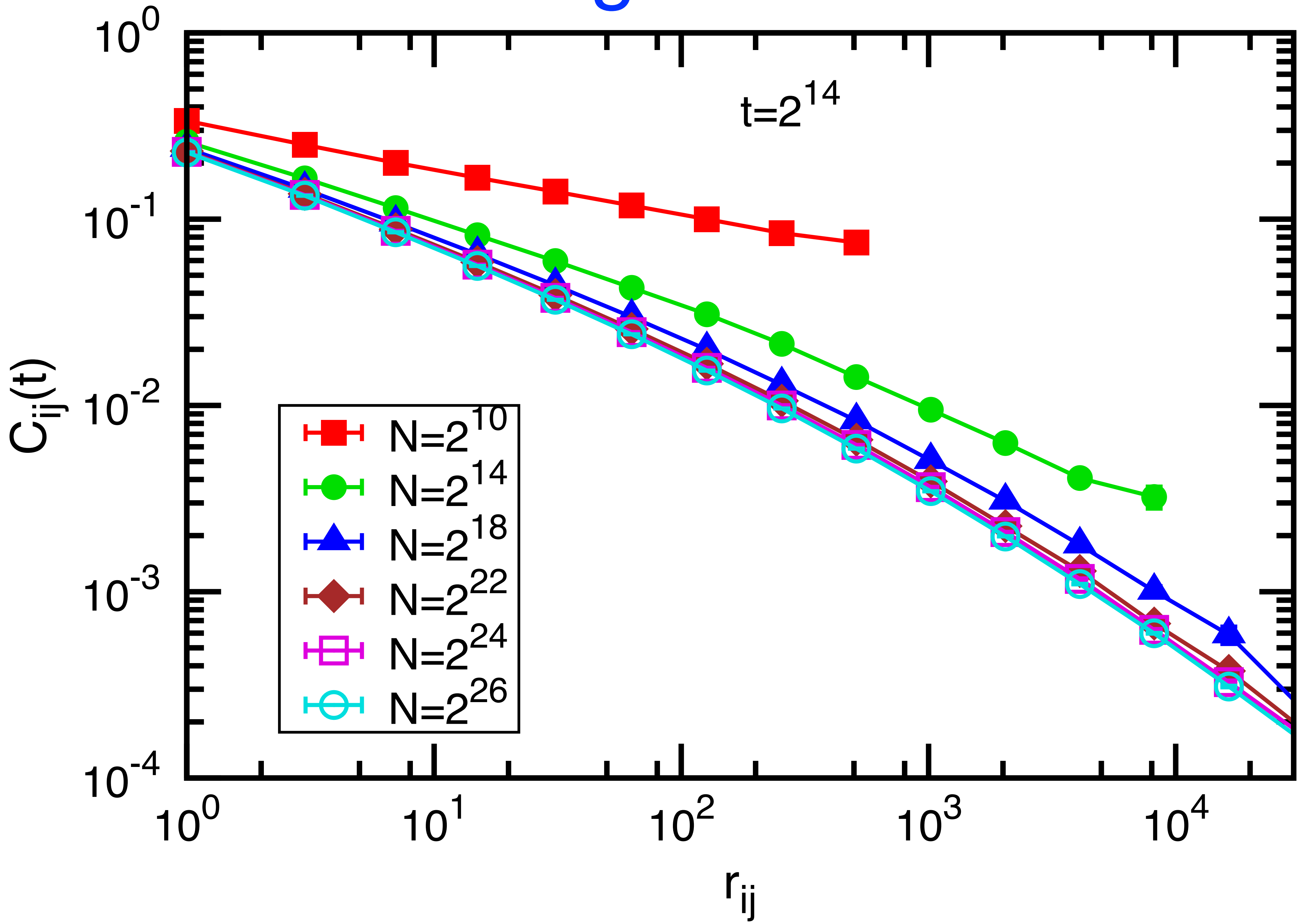
Hence, for the LR model $\alpha_s = \frac{d-4}{d} \; (= 3 - 4\sigma) = \frac{1}{2}$

Hence our goal is to see if **α_d = 1/2.**

We quench the system to $T = 0.4T_c = 0.74$ and then follow the time evolution.

Sizes: $N = 2^\ell$ for ℓ up to 26.
Times: $t = 2^k$ for k up to 14.

Convergence with size

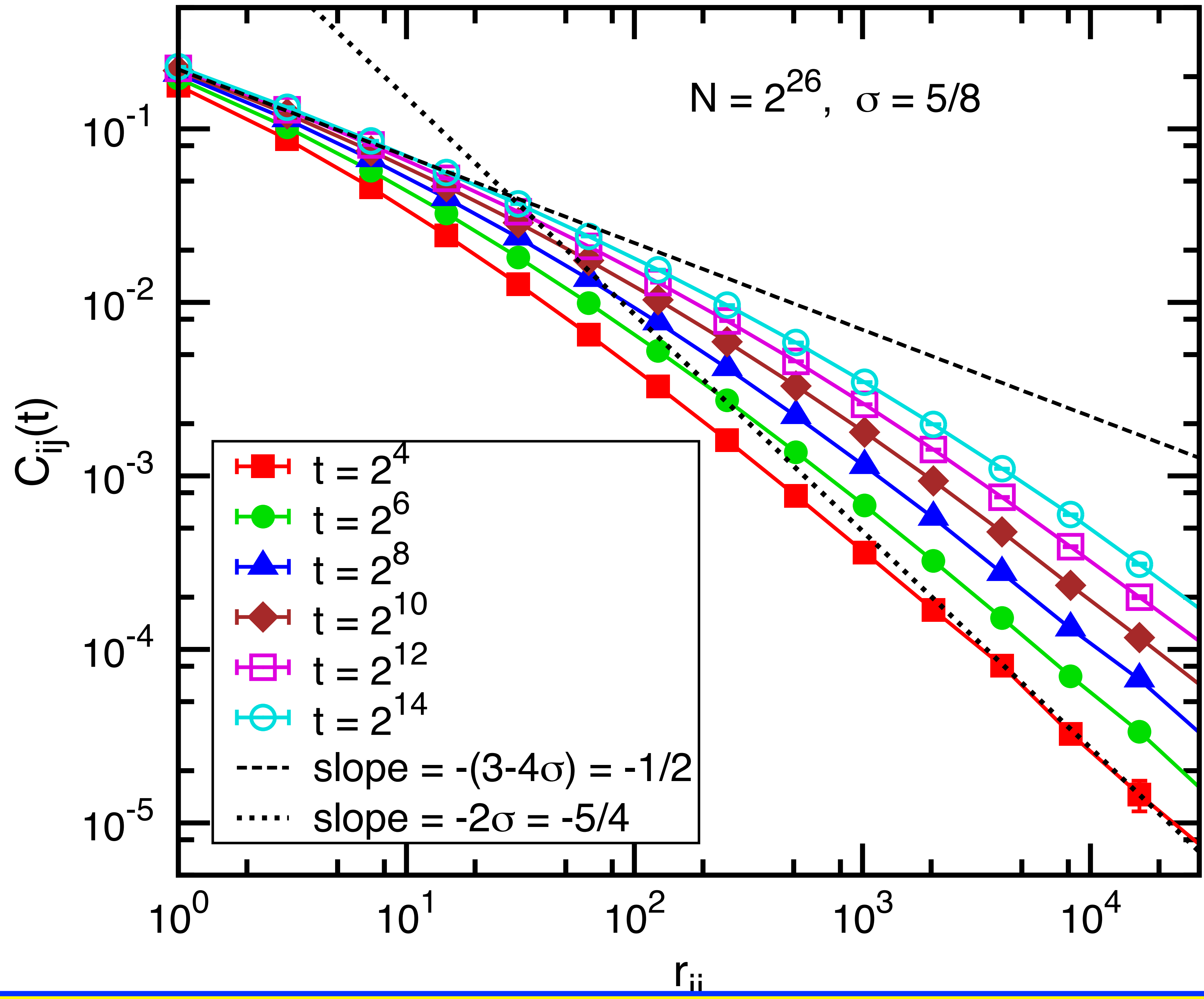


Data for different sizes for $t = 2^{14}$, the longest time we took.

The data for the largest sizes seems to have converged.

(Very large sizes were needed to overcome finite-size corrections.)

Crossover with r at different times

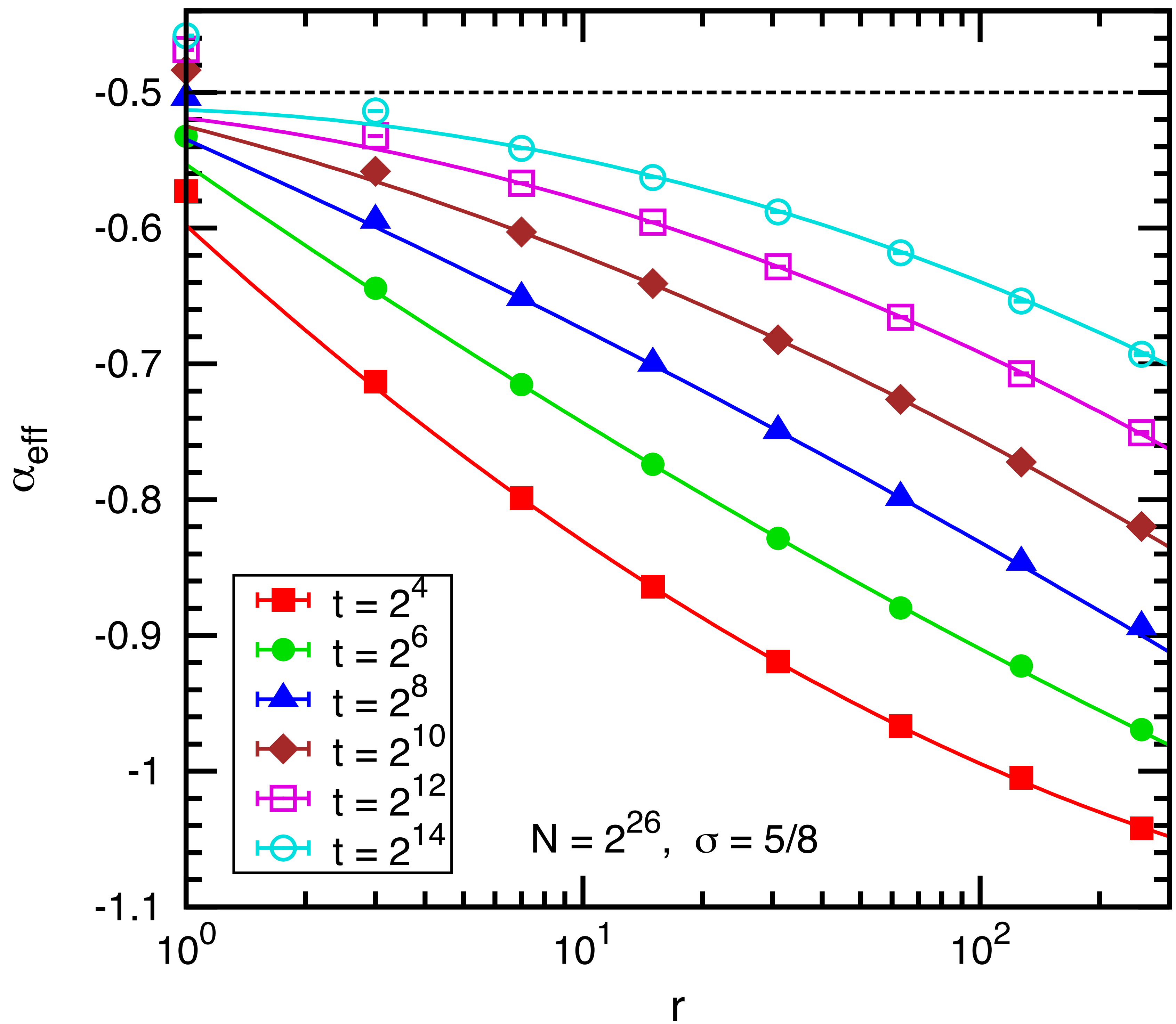


Data for $N = 2^{26}$, the largest size.
Plot shows data as a function of r for different times.

Small r, $r \ll \xi(t)$, slope is $\alpha_d = -(3 - 4\sigma) = -1/2$. Numerically same as α_s i.e. MAS

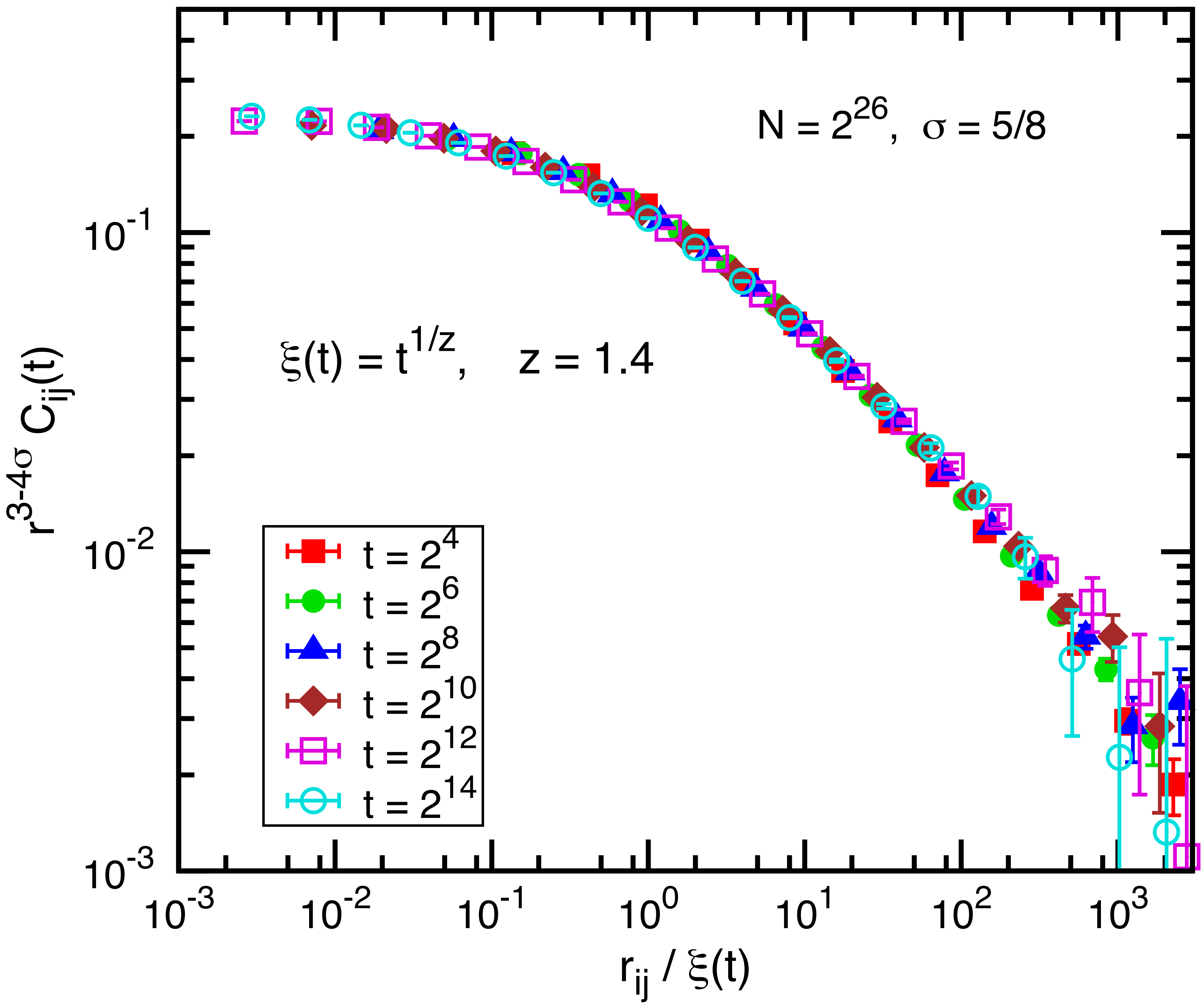
Large r, $r \gg \xi(t)$, slope is $-2\sigma = -5/4$ (due to LR interactions)

Effective α



The effective exponent α_{eff}, as a function of r. This is the slope of the curves in the previous figure, obtained by differentiating a spline fit.
The curves here are a quadratic fit for intermediate r, $7 \leq r \leq 255$.

Scaling collapse



Scaling plot of the data for $\xi(t) = t^{1/z}$ with $z = 1.4$.

If $z \propto 1/T$ then $z(T_c) = 0.56$.

Short-range: Corresponding SR value is d (=8) times as big, i.e. $z(T_c) = 4.5(6)$. Within errors this agrees with the MF critical value, $z_c = 4$ (Zippelius)

Conclusions

- Have investigated, in the mean field region, a possible connection between the dynamical and static metastates.
- Took a LR model corresponding to a SR model in $d = 8$ ($> d_u = 6$).
- Average over non-equilibrium dynamics gives same power-law behavior with r as the metastate average computed by Read assuming RSB.
- Are the dynamic and static metastates equivalent more generally?

Thank you