Study of Chirality in the Two-Dimensional XY Spin Glass

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We study the chirality in the Villain form of the XY spin glass in two–dimensions by Monte Carlo simulations. We calculate the chiral-glass correlation length exponent ν_{CG} and find that $\nu_{CG} = 1.8 \pm 0.3$ in reasonable agreement with earlier studies. This indicates that the chiral and phase variables are decoupled on long length scales and diverge as $T \to 0$ with *different* exponents, since the spin-glass correlation length exponent was found, in earlier studies, to be about 1.0.

Ever since Villain[1] pointed out the existence of a discrete reflection symmetry (in addition to the ordinary rotation) in frustrated vector spin systems, there has been considerable interest in the similarity or difference between the behavior the variables corresponding to these two symmetries - chiralities (which are quenched in vortices), corresponding to reflection, and spins, corresponding to rotation. In part this interest comes from the observation that spin glasses seem to be in the Ising universality class, though many of them should be described quite well by a Heisenberg model. This has led to speculations that chiralities and spins order differently and that the Ising behavior seen in experiments might indicate the existence of a chiral glass phase in the absence of spin glass ordering[2]. This view is supported by some numerical simulations[3,4]. However, in spite of many studies, the problem still remains controversial.

Kawamura and Tanemura[3] studied the two dimensional XY spin glass by a domain wall renormalization group technique and were the first to present evidence that the chiral and spin glass correlation length exponents are different in two-dimensions. They also reported Monte Carlo simulations which supported their claim[3]. A little later, Ray and Moore[4] also reported Monte Carlo results which indicated that in two-dimensions the chiral and spin glass correlation length exponents were indeed different. They estimated $\nu_{SG} \simeq 1.0$ and $\nu_{CG} \simeq 2.0$, the former result being in good agreement with earlier work of Jain and Young[5]. More recently, Kawamura[6] reported Monte Carlo simulations of the three dimensional XY spin glass and claimed that there is indeed a stable chiral glass phase. This would appear to have settled the issue. However, recent analytic work on the one-dimensional ladder lattice[7] and on the two-dimensional system with a special choice of disorder[8] points in the opposite direction. Given this controversy it seems a reasonable time to study the two-dimensional system once again numerically.

The earlier work used a representation of the model in terms of the phases of the XY model, see Eq. (2) below, and vortices are expressed in terms of correlations of the phases around an elementary square. However, vortices are only well defined when the nearest-neighbor spin-spin correlation function is large, which means that the temperature is already quite low. It is therefore difficult to study vortex correlations over a large range of temperature. In our work, we use a different representation of the model, expressed directly in terms of the vortices themselves. As a result it is possible to study the vortex correlations over a larger range of temperature than before.

The model used in the simulations is an XY spin glass in which the interactions, $J_{ll'}$, have values $\pm J$. The Hamiltonian is

$$\mathcal{H} = -\sum_{\langle l,l' \rangle} J_{ll'} \vec{S}_l \cdot \vec{S}_{l'} \ , \qquad (1)$$

where the $\vec{S}_i$ are two component vectors of unit length. This can also be written as

$$\mathcal{H} = -J \sum_{\langle l,l' \rangle} \cos(\phi_l - \phi_{l'} - A_{ll'}), \qquad (2)$$

where ϕ_l is the angle (phase) the XY spin makes with a fixed direction, and the $A_{ll'}$ take values 0 and π with equal probability, corresponding to $J_{ll'} = 1$ and -1 respectively. We take the sites, l, to lie on a square lattice of size $L \times L$, and the interaction is between all nearest neighbor pairs, counted once.

As discussed above, it is easier to study the model in terms of vortices. To do so we first replace the cosine in Eq. (2) by the Villain periodic Gaussian function, i.e.

$$\mathcal{H} = \sum_{\langle l,l' \rangle} V(\phi_l - \phi_{l'} - A_{ll'}), \qquad (3)$$

where

$$\exp\left(\frac{-V(x)}{T}\right) = \sum_{m=-\infty}^{\infty} \exp\left(\frac{-J(x-2\pi m)^2}{2T}\right) \ , \qquad (4)$$

in units where Boltzmann's constant is unity. Performing standard manipulations[9,10] one finds that the partition function of the Hamiltonian in Eq. (3) is the same (apart from an unimportant smoothly varying prefactor) as that of the following Hamiltonian,

$$\mathcal{H}_V = -\frac{1}{2} \sum_{i,j} (n_i - b_i) G(i-j) (n_j - b_j), \qquad (5)$$

where the vortices $\{n_i\}$ run over all integer values, subject to the "charge neutrality" constraint

$$\sum_i n_i = 0 \ , \tag{6}$$

and $G(i-j)$ is the vortex interaction,

$$\frac{G(i-j)}{(2\pi)^2} = \frac{J}{N}\sum_{\mathbf{k}\neq 0} \frac{1-\exp[i\mathbf{k}\cdot(\mathbf{r_i}-\mathbf{r_j})]}{4-2\cos k_x - 2\cos k_y} \quad . \tag{7}$$

Note that the Fourier transform of the vortex interaction is $\sim k^{-2}$ for small k which corresponds to a long range logarithmic interaction in real space. The vortices sit on the sites i of the dual lattice which are in the centers of the squares of the original lattice. The b_i are given by $(1/2\pi)$ times the directed sum of the quenched random variables, $A_{ll'}$, on the links of the original lattice which surround the site i of the dual lattice. They satisfy a constraint similar to Eq. (6),

$$\sum_i b_i = 0 \ . \tag{8}$$

From now on we set the interaction strength, J, to be unity.

For $A_{ll'} = 0$ or π there are two kinds of plaquettes on the original lattice, or equivalently two kinds of sites on the dual lattice: there are unfrustrated sites, on which b_i is integer, and frustrated sites, on which b_i is half-integer. If $A_{ll'}$ has equal probability to takes values 0 and π then half the sites on the dual lattice will be frustrated and half unfrustrated on average. In the ground state, most unfrustrated sites will have $n_i = 0$, while most frustrated sites will have $n_i = \pm 1/2$. These Ising-like variables are precisely the chirality variables of Villain. It seems reasonable that the essential physics will be preserved if one just keeps these chirality variables, i.e. one fixes n_i to be zero on the unfrustrated sites and only allows n_i to be $\pm 1/2$ on the frustrated sites. Thus one has an Ising model with a long range antiferromagnetic coupling,

$$\mathcal{H}_{CG} = -\frac{1}{2}\sum_{i,j} \tilde{G}(i-j)\sigma_i\sigma_j\epsilon_i\epsilon_j \tag{9}$$

where, for convenience, we represent the chiralities by Ising spins, σ_i, of *unit* length so $\sigma_i = \pm 1$, and the interaction, $\tilde{G}$, is then equal to $G/4$. The quenched variable ϵ_i is equal to 1 if there is a chirality on site i, and is equal to 0 if there is no chirality. The lattice sites with chirality are to be chosen at random with 50% probability. We impose the additional contraint that the total number of chiralities is *exactly* half the number of lattice sites. Defining N to be the number of chiralities we have, for *every* sample,

$$N = \frac{L^2}{2}. \tag{10}$$

We now discuss the scaling theory and the details of our Monte Carlo simulations. We run two independent replicas of the system in parallel with the same realization of the disorder and compute the overlap between the states in the two replicas,

$$q = \frac{1}{N}\sum_{i=1}^{N} \sigma_i^{(1)}\sigma_i^{(2)} \ . \tag{11}$$

Here, and in the rest of the paper, the sum over sites i and j on the dual lattice is only over those sites occupied by a chirality. The standard spin glass order parameter is just $[\langle q\rangle]$, where the angular brackets denote the thermal averages and the square brackets the average over disorder. Two useful quantities are the Binder moment ratio g_{CG} and the chiral glass susceptibility χ_{CG} defined by

$$g_{CG} = \frac{1}{2}\left\{3 - \frac{[\langle q^4\rangle]}{[\langle q^2\rangle]^2}\right\} \tag{12}$$

and

$$\begin{aligned}\chi_{CG} &= \frac{1}{N}\sum_{i,j}[\langle\sigma_i\sigma_j\rangle^2] \\ &= N[\langle q^2\rangle] \ .\end{aligned} \tag{13}$$

g_{CG} is defined so that it tends to 0 at high temperatures in the thermodynamic limit, and tends to unity as $T \to 0$ if the ground state is non-degenerate. The chiral glass susceptibility should be contrasted with the spin glass susceptibility, χ_{SG}, defined by

$$\begin{aligned}\chi_{SG} &= \frac{1}{L^2}\sum_{l,l'}[\langle\cos(\theta_l - \theta_{l'})\rangle^2] \\ &= L^2\sum_{\alpha,\beta}[\langle q_{\alpha\beta}^2\rangle] \ ,\end{aligned} \tag{14}$$

where

$$q_{\alpha\beta} = \frac{1}{L^2}\sum_{l=1}^{L^2} S_{l,\alpha}^{(1)} S_{l,\beta}^{(2)} \ . \tag{15}$$

Here α and β denote the components of the XY spins in Eq. (1) and take values x and y.

Since the transition in this system is expected to be at $T = 0$ the following scaling forms are expected for χ_{CG} and for g_{CG}[12]:

$$g_{CG} = \tilde{g}_{CG}(L^{1/\nu_{CG}}T) \tag{16}$$

and

$$\chi_{CG} = L^{2-\eta_{CG}}\tilde{\chi}_{CG}(L^{1/\nu_{CG}}T) \ . \tag{17}$$

Here the exponent ν_{CG} is the correlation length exponent and η_{CG} is related to the ground state degeneracy—if the ground state is unique, one expects $\eta_{CG} = 0$. At a critical point, the data for g_{CG} should be independent of size. This is a particularly convenient way of locating the

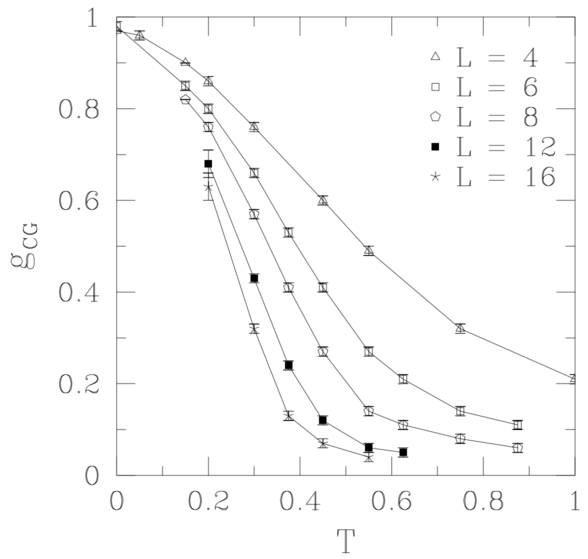

FIG. 1. Results for the Binder moment ratio g_{CG}, defined in Eq. (12), for different sizes and temperatures. The lines are guides to the eye.

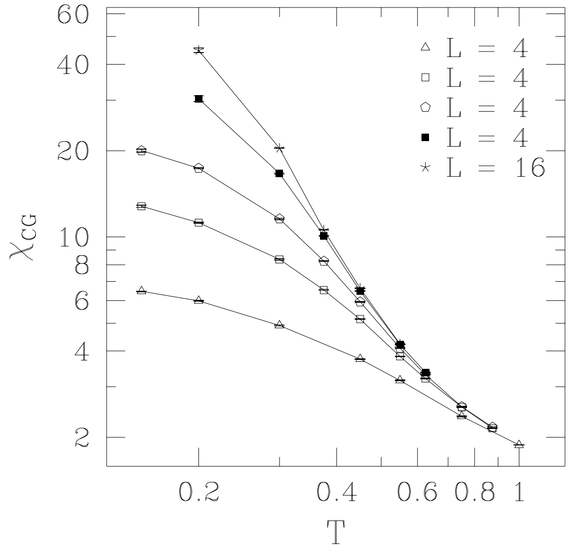

FIG. 2. A log-log plot of results for the chiral glass susceptibility χ_{CG}, defined in Eq. (13), for different sizes and temperatures. The lines are guides to the eye.

transition. The spin glass susceptibility in Eq. (14) has the finite size scaling form

$$\chi_{SG} = L^{2-\eta_{SG}} \tilde{\chi}_{SG}(L^{1/\nu_{SG}} T) \ , \tag{18}$$

with exponents ν_{SG} and η_{SG} which are expected to be *different* from the corresponding chiral glass exponents. In this paper, we just focus on the chiral glass critical behavior.

We use standard methods[12] to ensure equilibration of the Monte Carlo simulation. Various quantities are computed both from overlaps between the two replicas and from a single replica at different times, see Bhatt and Young[12] for details. Typically, for the largest lattice sizes we needed about 100000 Monte Carlo sweeps for equilibration at the lowest temperature. For the averaging over disorder we took between 1000 and 10000 samples. For most of the data points the statistical errors were estimated by averaging over all the samples. However, for the largest lattice sizes and lowest temperatures we divided up the samples into blocks of a few hundred each and calculated the errors from the standard deviations of the quantities between different blocks.

The results for g_{CG} as a function of temperature for different sizes are shown in Fig. 1 and the corresponding results for χ_{CG} are in Fig. 2. The points at $T = 0$ are obtained by exact enumeration of all the states.

We also calculated the ground state degeneracy for L = 4 and L = 6 and found that there is a small non-zero degeneracy, which leads to g_{CG} being slightly below 1 at $T = 0$. However, g_{CG} increases slightly with increasing size, so we expect that g_{CG} will be unity at $T = 0$ in the thermodynamic limit. One should note here that Ray and Moore[4], who worked in the phase representation, also found that g_{CG} is not very sensitive to the ground state degeneracy, and seems to extrapolate to unity.

In order to do a finite size scaling analysis for g_{CG} it is convenient to incorporate one trivial correction to scaling. As defined in Eq. (12), g_{CG} tends to $1/N$ (rather than 0) as $T \to \infty$, which is not completely negligible for the sizes studied here. We therefore consider the following quantity

$$g'_{CG} \equiv \frac{N g_{CG} - 1}{N - 1} = \tilde{g}_{CG}(L^{1/\nu_{CG}} T) \ , \tag{19}$$

which does vanish at high temperatures for finite N (and tends to unity as $T \to 0$ if g_{CG} does).

Fig. 3 shows our finite size scaling plot for g'_{CG}, including data for $T \le 0.55$. Data at higher temperatures did not scale well and is presumably not in the scaling region. It is clear that good data collapse obtains with $\nu_{CG} = 2.0$. We tried scaling with other values of ν_{CG} and found that data collapse gets worse both for $\nu_{CG} > 2.0$ and for $\nu_{CG} < 2.0$. Trying various values for ν_{CG} in this way we estimate $\nu_{CG} = 2.0 \pm 0.2$ from the data for g_{CG} in Fig. 3.

Just at it is convenient to put in the correction to g_{CG} in Eq. (19) in order that it varies between 1 and 0 as the temperature changed, it is also useful to perform a similar transformation for χ_{CG}. Since χ_{CG} tends to unity as $T \to 0$, we subtract unity. Furthermore, our zero temperature data is consistent with η_{CG} very close to zero so, from now on, we will assume that $\eta_{CG} = 0$ and

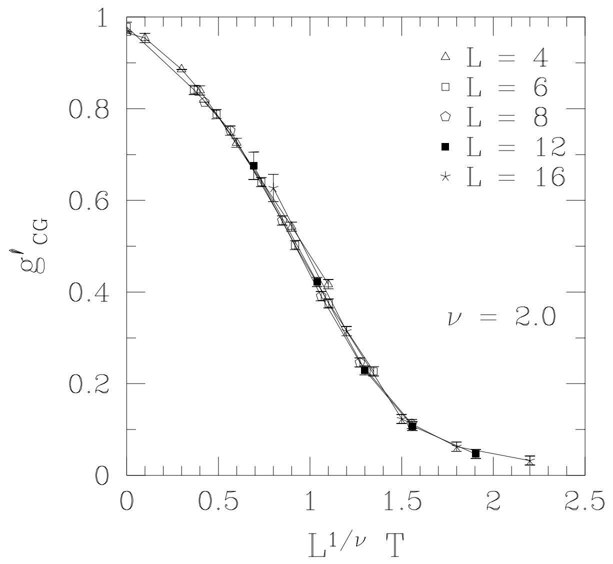

FIG. 3. A scaling plot of the data in Fig. 1 according to Eq. (19). Data for $T \le 0.55$ has been included.

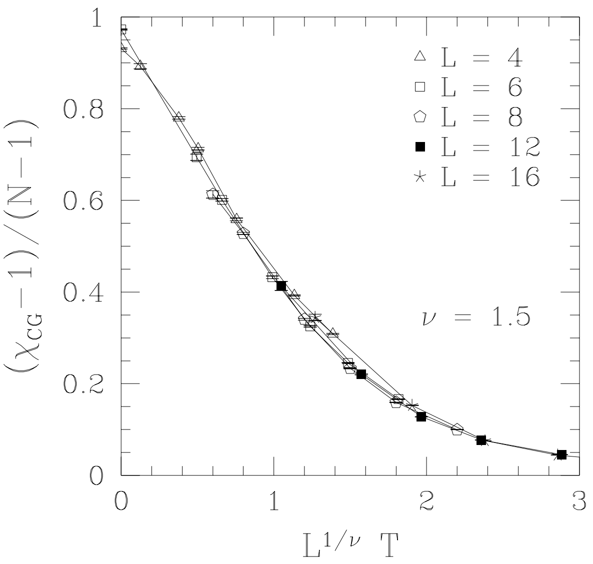

FIG. 4. A scaling plot of the data in Fig. 2 according to Eq. (20) with $\nu_{CG} = 1.5$. The data used in the plot is for $T \le 0.55$. This plot *assumes* $\eta_{CG} = 0$, which is reasonable since the $T = 0$ results indicate that η_{CG} is close to, and probably exactly equal to, zero.

that $\chi_{CG} = N$ at $T = 0$ in the thermodynamic limit. Hence we will analyze

$$\chi'_{CG} \equiv \frac{\chi_{CG} - 1}{N - 1} = \tilde{\chi}_{CG}(L^{1/\nu_{CG}} T) \ , \tag{20}$$

which varies between 1 and 0 as T increases.

In Fig. 4 we show a finite size scaling plot for χ'_{CG}, including data for $T \le 0.55$. The best fit is for $\nu_{CG} = 1.5$, somewhat lower than that obtained from g_{CG}. A similar difference is also found in the three-dimensional Ising spin glass[13].

Combining our exponent estimates from g_{CG} and χ_{CG} we obtain

$$\begin{aligned} \nu_{CG} &= 1.8 \pm 0.3 \\ \eta_{CG} &= 0.0 \pm 0.2 \ . \end{aligned} \tag{21}$$

These results indicate that the chiralities in the two-dimensional XY spin glass order with a correlation length exponent ν_{CG} which is different from the spin glass correlation length exponent, for which earlier work[4,5] found $\nu_{SG} \simeq 1$. This conclusion is in agreement with earlier results[3,4]. For the future it would be useful to study the *three*-dimensional XY spin glass in the vortex representation to understand whether there is indeed a finite temperature chiral-glass phase as found in the work of Kawamura[6].

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