Monte Carlo Study of a Three–Dimensional Vortex Glass Model with Screening

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We investigate the critical behavior of the gauge glass model for the vortex glass transition in three-dimensional superconductors, including screening of the interaction between vortices. A Monte Carlo study of the linear resistivity and a scaling analysis of current–voltage characteristics indicates that screening destroys the finite–temperature transition found earlier when screening was neglected. The correlation length exponent at the resulting zero temperature transition is found to be $\nu = 1.05 \pm 0.1$.

The question of whether type–II superconductors in the mixed state exhibit a finite linear resistance, $\rho_{\rm lin}$, in the limit of a vanishingly small current has been under extensive consideration in recent years. While an applied current perpendicular to the field generates a Lorentz force and hence leads to motion of flux lines producing a voltage[1], it has been proposed by Fisher[2] that disorder (point defects) collectively pins the vortices, resulting in a phase with vanishing linear resistance. Since disorder also destroys the triangular vortex lattice predicted by mean field theory[3], this phase is called the *vortex glass*.

Much of the theoretical work has been devoted to the study of a simplified model, called the gauge glass, which has the necessary ingredients of randomness and frustration and is believed to be in the same universality class as the vortex glass. In two dimensions, experiments[4], and simulations[5–7] agree that the vortex glass transition only occurs at $T_c = 0$. In $d = 3$ the situation is somewhat less clear. Several experiments on YBCO[8] have found convincing evidence for a finite T_c phase transition, presumably to a vortex glass. Furthermore, simulations support a transition at $T_c > 0$, although the possibility that $T_c = 0$ and the lower critical dimension is precisely $d = 3$ cannot be fully ruled out[9,7].

Most of the theoretical work on the vortex glass has neglected screening of the interaction between vortices, which arises from coupling to the fluctuating magnetic field. However, sufficiently close to T_c the screening length, λ, and the correlation length, ξ, become comparable and the system crosses over into a region where screening is important[7,10]. A recent domain wall renormalization study of the $d = 3$ gauge glass model by one of us[7] (henceforth referred to as BY) concluded that screening apparently destroys the finite temperature transition. For experimental systems this effect would lead to a rounding out of the transition close to the nominal T_c, and the linear resistance would not vanish completely but become very small. So far, experiments have not had the sensitivity to see this effect. However, the calculations of BY were on very small lattice sizes, $L \le 4$, and so their conclusions can only be tentative. In this paper, we investigate the role of screening by Monte Carlo simulations, for which much larger sizes are possible, $L \le 12$. Our results confirm the conclusion of BY that screening destroys the finite–T transition. We are also able to obtain, for the first time, values for the exponents at the resulting $T = 0$ transition.

The model studied here is the gauge glass with a fluctuating vector potential

$$\mathcal{H} = -J \sum_{\langle i,j\rangle} \cos(\phi_i - \phi_j - A_{ij} - \lambda_0^{-1} a_{ij}) + \frac{1}{2} \sum_{\square} [\vec{\nabla} \times \vec{a}]^2, \tag{1}$$

where J is the interaction strength, ϕ_i is the phase of the condensate on site i, and the sum is over all nearest neighbors $\langle i, j\rangle$ on a simple cubic lattice. The effects of the external field and disorder are represented by quenched vector potentials A_{ij}, which we take to be uniformly distributed on the interval $[0, 2\pi]$. The fluctuating vector potentials a_{ij} are integrated over from $-\infty$ to ∞, with a gauge constraint $\vec{\nabla} \cdot \vec{a} = 0$, and λ_0 is the bare screening length. In the last term, which describes the magnetic energy, the sum is over all elementary squares on the lattice and the curl is the directed sum of the vector potentials round the square.

We consider the strong screening limit, $\lambda_0 \to 0$, which is technically easier to simulate in the vortex representation. This is done by replacing the cosine in Eq. (1) with the periodic Villain function and performing fairly standard manipulations[11–13,7] to obtain

$$\mathcal{H}_V = -\frac{1}{2} \sum_{i,j} G(i - j)\, [\mathbf{n}_i - \mathbf{b}_i] \cdot [\mathbf{n}_j - \mathbf{b}_j]. \tag{2}$$

Here the $\mathbf{n}_i$ are vortex variables which sit on the links of the *dual* lattice (which is also a simple cubic lattice), $G(i - j)$ is the screened lattice Green's function

$$G(i - j) = J \frac{(2\pi)^2}{L^3} \sum_{\mathbf{k}\neq 0} \frac{1 - \exp[\mathrm{i}\, \mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)]}{2 \sum_{n=1}^{d} [1 - \cos(k_n)] + \lambda_0^{-2}}, \tag{3}$$

(with $d = 3$), and the $\mathbf{b}_i$ are given by $(1/2\pi)$ times the directed sum of the quenched vector potential on the original lattice surrounding the link on the dual lattice on which $\mathbf{b}_i$ lies. Due to periodic boundary conditions, we have the global constraints $\sum_i \mathbf{b}_i = \sum_i \mathbf{n}_i = 0$. There are also the *local* constraints, $[\nabla \cdot \mathbf{n}]_i = [\nabla \cdot \mathbf{b}]_i = 0$, where the latter just follows trivially from the definition of $\mathbf{b}_i$ as a lattice curl.

Clearly $G(0) = 0$ and, in the limit $\lambda_0 \to 0$, $G(r) = J(2\pi\lambda_0)^2$ for $r \neq 0$ with corrections which are exponentially small, *i.e.* of order $\exp(-r/\lambda_0)$. Because

$\sum_i(\mathbf{b}_i - \mathbf{n}_i) = 0$ we can always add a constant to $G(r)$ for all r without affecting the results. We therefore add $-(2\pi\lambda_0)^2 J$, as a result of which the only interaction is on-site, and then work in units where $(2\pi\lambda_0)^2 J$ is unity. The resulting Hamiltonian then has the very simple form

$$\mathcal{H}_V = \frac{1}{2}\sum_i (\mathbf{n}_i - \mathbf{b}_i)^2, \tag{4}$$

which is convenient for simulations. Note, however, that $\mathcal{H}_V$ is not trivial because the local constraint $[\nabla \cdot \mathbf{n}]_i = 0$ effectively generates interactions between the $\mathbf{n}_i$. Note also that without disorder (*i.e.* all $\mathbf{b}_i = 0$) this model is just a dual representation of the XY–model[12,13] (with the Villain potential), in which the temperature scale has been inverted.

We simulate the Hamiltonian in Eq. (4) on simple cubic lattices with $N = L^3$ sites where $4 \le L \le 12$. Periodic boundary conditions are imposed. We start with configurations with all $\mathbf{n}_i = 0$, which clearly satisfies the constraints, and a Monte Carlo move consists of trying to create a loop of four vortices around a square. This trial state is accepted with probability $1/(1+\exp(\beta\Delta E))$, where ΔE is the change of energy and $\beta = 1/T$. Each time a loop is formed it generates a voltage $\Delta Q = \pm 1$ perpendicular to it's plane, the sign depending on the orientation of the loop. This leads to a net voltage[6]

$$V(t) = \frac{h}{2e} I^V(t) \quad \text{with} \quad I^V(t) = \frac{1}{L\Delta t}\Delta Q(t), \tag{5}$$

where I^V is the vortex–current and t denotes Monte Carlo "time" incremented by Δt for each attempted Monte Carlo move. We will work in units where $h/(2e) = 1$, and we set $\Delta t = 1/3N$ so that an attempt is made to create or destroy one vortex loop per square in each direction, on average, per unit time.

The linear resistivity can be calculated from the voltage fluctuations via the Kubo formula[14]

$$\rho_{\rm lin} = \frac{1}{2T}\sum_{t=-\infty}^{\infty} \Delta t \, \langle V(t)V(0)\rangle. \tag{6}$$

Near a second order phase transition the linear resistivity obeys the scaling law[10]

$$\rho_{\rm lin}(T, L) = L^{-(2-d+z)}\tilde{\rho}(L^{1/\nu}(T - T_c)), \tag{7}$$

where $\tilde{\rho}$ is a scaling function and z is the dynamical exponent. At the critical temperature, $\tilde{\rho}$ becomes a constant and therefore $\rho_{\rm lin}(T_c, L) \sim L^{-(2-d+z)}$. If we plot the ratio $\ln[\rho_{\rm lin}(L)/\rho_{\rm lin}(L')]/\ln[L/L']$ against T, then at the point $(T_c,\, d-2-z)$ all curves for different pairs (L, L') should intersect and one can read off the values of T_c and z. We will refer to this kind of data plot as the "intersection method".

In addition to $\rho_{\rm lin}$, we also measure the voltage generated by a finite external current. In real superconductors, transport currents generate a non-uniform magnetic field because of Ampère's law, $\vec{\nabla} \times \mathbf{B} = \mathbf{J}$. It is inconvenient to simulate a non–uniform system, so instead we effectively assume that the current is the same everywhere so *each* vortex feels a Lorentz force $\mathbf{n}_i \times \mathbf{J}$. The scaling behavior of the response to such a perturbation should be the same as that derived earlier for response to an actual transport current[10]. We can therefore use this approach to determine critical exponents, which is our objective. The Lorentz force biases the moves and sets up a net flow of vortices perpendicular to the current, whose time average gives the voltage according to[6] Eq. (5).

To analyze our data we need to understand the scaling behavior of the I-V–curves near a second order phase transition. We quickly review the scaling theory[10] assuming $T_c = 0$ so the correlation length diverges as $\xi \sim T^{-\nu}$. The vector potential enters the Hamiltonian in Eq. (1) in the dimensionless form $A_{ij} = \int_i^j \mathbf{A}(\mathbf{r}) \cdot d\mathbf{r}$ so $\mathbf{A}$ scales as $1/\xi$. The electric field is given by $\mathbf{E} = -\partial_t \mathbf{A}$ and so scales as $1/(\xi\tau)$ where τ is the relaxation time. $\mathbf{J} \cdot \mathbf{E}$ is the energy dissipated per unit volume and unit time and scales as[15] $T/(\xi^d\tau)$. Hence J scales like T/ξ^{d-1}. Combining these results leads to

$$T\frac{E}{J}\frac{\tau}{\xi^{d-2}} = g\left(\frac{J\xi^{d-1}}{T}\right), \tag{8}$$

where g is a scaling function. In three dimension this becomes

$$T^{1+\nu}\frac{E}{J}\tau = g\left(\frac{J}{T^{1+2\nu}}\right). \tag{9}$$

From this equation we can see that the current scale, J_{NL}, at which nonlinear behavior sets in varies with T as $J_{\rm NL} \sim T^{1+2\nu}$. Since the linear resistivity is defined by

$$\rho_{\rm lin} = \lim_{J\to 0}\frac{E}{J}, \tag{10}$$

and $g(0)$ can be taken to be unity, we can write

$$\frac{E}{J\rho_{\rm lin}} = g\left(\frac{J}{T^{1+2\nu}}\right). \tag{11}$$

Furthermore, we expect that near the $T = 0$ transition, long time dynamics will be governed by activation over barriers. Hence we expect

$$T^{1+\nu}\rho_{\rm lin} = \frac{1}{\tau} = A\exp(-\Delta E(T)/T), \tag{12}$$

where ΔE is the typical barrier that a vortex has to cross to move a distance ξ. One can define a barrier height exponent ψ by $\Delta E \sim \xi^\psi \sim T^{-\psi\nu}$ in terms of which

$$T^{1+\nu}\rho_{\rm lin} = A\exp(-C/T^{1+\psi\nu}). \tag{13}$$

We are able to obtain a rough estimate for ψ from our data of the linear resistivity.

In a finite system, the I-V characteristics will also depend on the ratio L/ξ. One can generalize the scaling function, Eq. (11), to account for finite size effects as follows:

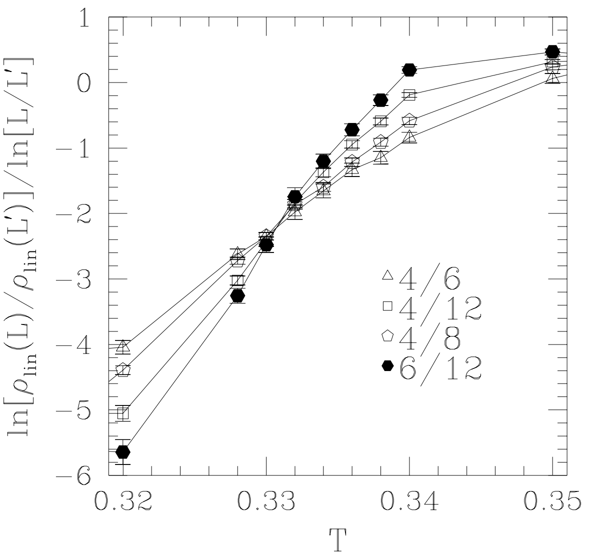

FIG. 1. Plot of $\ln[\rho_{\rm lin}(L)/\rho_{\rm lin}(L')]/\ln[L/L']$ versus T for the pure system, whose Hamiltonian is given by Eq. (4) with $\mathbf{b}_i = 0$. This is a dual representation of the pure XY model (with the Villain potential) with inversion of the temperature scale. The curves intersect at $T_c = 0.331 \pm 0.002$. At the intersection point, the y–value is approximately -2 corresponding to $z \simeq 3$ according to Eq. (7).

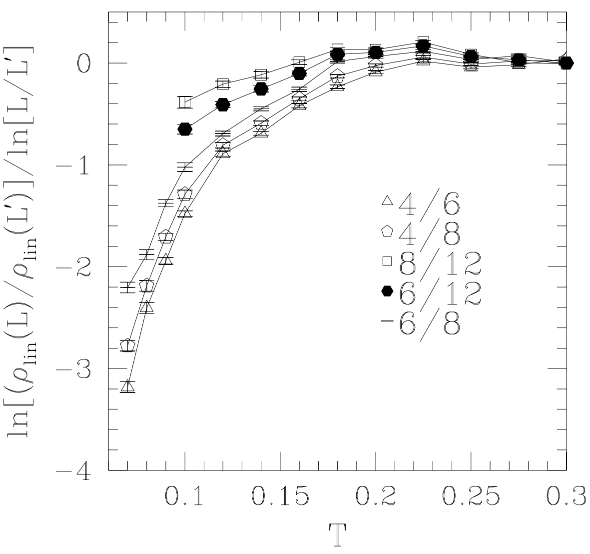

FIG. 2. Plot of $\ln[\rho_{\rm lin}(L)/\rho_{\rm lin}(L')]/\ln[L/L']$ against T for the gauge glass model whose Hamiltonian is given by Eq. (4). In contrast to the data for the pure system in Fig. 1, there is no intersection over the entire temperature range, indicating the absence of a phase transition.

$$\frac{E}{J\rho_{\rm lin}} = \tilde{g}\left(\frac{J}{T^{1+2\nu}}, L^{1/\nu}T\right). \tag{14}$$

Now we are left with a rather complicated scaling function since it depends on two variables. We estimated ν by determining the current where $E/(J\rho_{\rm lin}) = 2$, at which point non-linear effects start to become significant. Denoting these values of J by $J_{\rm NL}$, then, from Eq. (14), it follows that $J_{\rm NL}/T^{1+2\nu}$ is a function of $L^{1/\nu}T$. Collapsing the data in the appropriate plot then gives an estimate of ν. We then collected data for sizes and temperatures such that $L^{1/\nu}T$ is constant so the scaling function in Eq. (14) then depends only on one variable.

Let us now move to the analysis of our data. In Fig. 1 we show data of the linear resistivity for the pure case plotted according to the intersection method. All curves intersect at about $T = 0.331 \pm 0.002$ which is in excellent agreement with the well established value[12,11] of $T_c = 0.33$. The y–axis value at the intersection point is $1 - z$ according to Eq. (7), from which we estimate $z \simeq 3$. This is larger than one would expect for a model with relaxation dynamics where usually $z \simeq 2$. Perhaps corrections to finite size scaling make the estimates of T_c and z slightly inaccurate, with the effect on z being more pronounced since the slopes of the curves in Fig. 1 are quite steep. Nonetheless, we are interested in whether or not a transition occurs rather than precise values of exponents, and the method seems to be a reliable in predicting this, at least for the pure case.

For comparison, Fig. 2 shows the intersection method applied to the gauge glass model. As one can see, there is no apparent intersection over the entire temperature range that we have been able to simulate, *i.e.* down to $T = 0.1$ for $L = 12$ and down to $T = 0.07$ for $L \leq 8$. This rules out a transition down to about 1/5 of the critical temperature of the pure system, and strongly suggests that the phase transition is at zero temperature.

Fig. 3 shows a scaling plot of the nonlinear I-V characteristics according to Eq. (14). From the scaling of $J_{\rm NL}$ we estimated ν to be approximately 1.1, and then chose temperatures and sizes for the data in Fig. 3 such that $L^{1/\nu}T$, and hence the second argument in Eq. (14), remained roughly constant. The data is seen to scale very well. Combining this data together with scaling plots for other sizes and temperatures we obtained the overall estimate $\nu = 1.05 \pm 0.1$. We also tried to scale our data with an appropriate scaling function for finite T_c and found that scaling works satisfactorily with $T_c \simeq 0.04$ and $\nu \simeq 1.1$. Therefore, we conclude that the transition is very likely to occur at $T_c = 0$, although a finite but extremely small T_c cannot be completely ruled out.

Finally, Fig. 4 shows data for $T^2\rho_{\rm lin}$ versus $1/T$ in a log–linear plot according to Eq. (13). The data for $L = 12$ follows almost a straight line, *i.e.* shows close to Arrhenius behavior, which corresponds to a barrier exponent $\psi = 0$ according to Eq. (13). It is therefore possible that the barriers only increase logarithmically as the temperature approaches zero. However, such behavior is difficult to observe in finite size systems over a modest range of temperatures. It is also possible that we are only measuring an effective exponent and that the

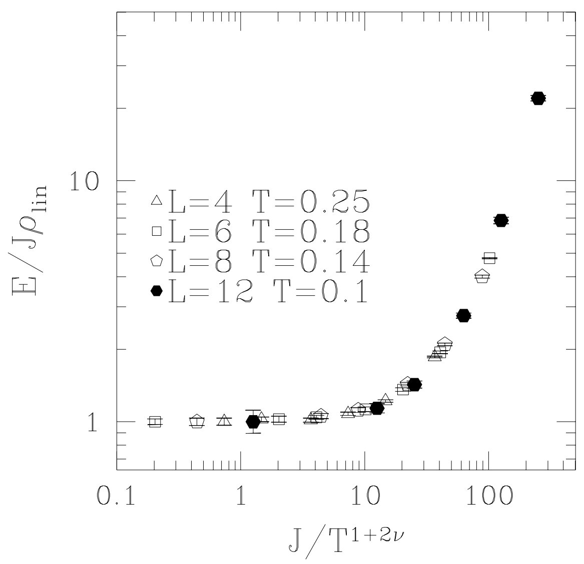

FIG. 3. Scaling plot of the I-V–characteristics with $T_c = 0$ and $\nu = 1.05$, according to Eq. (14), choosing sizes and temperatures such that $L^{1/\nu}T$ is roughly constant.

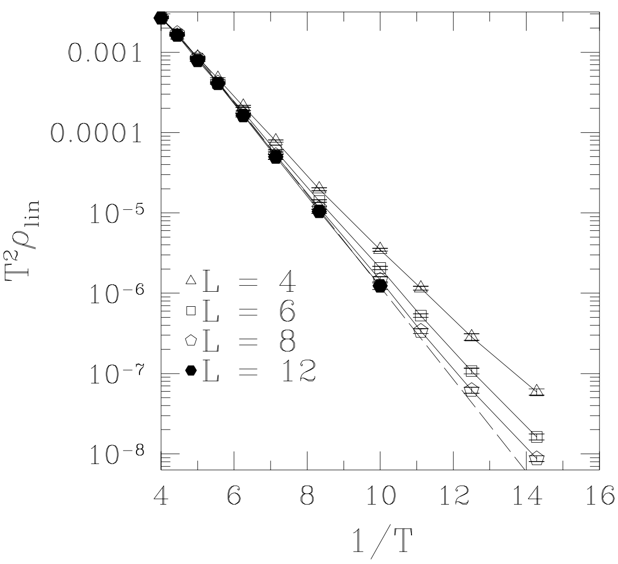

FIG. 4. Plot of $T^2\rho_{\rm lin}$ a logarithmic scale versus $1/T$. The dashed line corresponds to an Arrhenius law, which, according to Eq. (13), corresponds to a barrier exponent, ψ, equal to zero. The data for the largest size, $L = 12$, is well approximated by this.

true ψ is larger than zero.

In summary, we have performed a Monte Carlo study of the $d = 3$ gauge glass model in the vortex representation with strong screening. We have analyzed the linear resistivity and I-V characteristics by means of finite size scaling. There appears to be no transition of this model into a vortex glass state at finite temperature — consistent with the findings of BY — and the correlation length exponent at the resulting zero temperature transition is found to be $\nu = 1.05 \pm 0.1$. However, the possibility that T_c is small but non–zero cannot be completely ruled out since we are not able to equilibrate a range of system sizes below about 1/5 of the transition temperature of the pure system. Assuming $T_c = 0$, the presumed finite temperature vortex glass transition in the three–dimensional gauge glass is destroyed by screening effects. This means that the linear resistance in experimental systems would not strictly vanish, though it would become extremely small for T near the apparent T_c. It would be interesting to look for this effect experimentally.

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[15] Note that it is important to keep the factor of T for a zero temperature transition.