

## PHYSICS 115/242

### Homework 6

Due in class, Monday May 19.

#### A start with Mathematica

Other than possibly question 3, which is in C, please put the solutions in a single Mathematica notebook (and print this out). You can do explanations in Mathematica cells formatted as text. You can insert question numbers by a cell formatted as a subsection. If you are unsure please ask me (and *not at the last minute!*)

1. Give the *exact* value of 200!
2. Give the value of 50000! in floating point notation to at least 5 decimal places.  
*Note:* I don't advise you to try to print out the exact value.
3. Determine 50000! in floating point notation to at least 5 decimal places using a C or Fortran program.  
*Note:* Compute  $\log 50000!$  (remember that the log of the product is the sum of the logs.) (Don't use Stirling's approximation). From the log of 50000! you should be able to deduce the exponent and mantissa of 50000! without overflow problems. However, Mathematica is clearly easier.
4. Is  $\exp(\pi\sqrt{163})$  an integer? How many digits of precision do you need to determine this?  
*Note:* You may find it convenient to use `AccountingForm[...]` (or equivalently `//AccountingForm` as a "postfix" command) which causes the result to be printed in fixed point, rather than floating point, decimal format (i.e. no powers of 10).
5. Solve numerically for all the roots of the equation

$$x^7 + x^5 + 2x^3 + x^2 + 1 = 0.$$

6. Evaluate numerically the integral

$$\int_{-\infty}^{\infty} H_4(x)^2 e^{-x^2} dx$$

where  $H_4(x)$  is a Hermite polynomial (arises in the solution of the simple harmonic oscillator in quantum mechanics). The Mathematica function for  $H_n(x)$  is `HermiteH[n, x]`.

7. Solve numerically the differential equation

$$\frac{d^2y}{dx^2} = 2x + y + 3\frac{dy}{dx},$$

in the range  $2 \leq x \leq 2.3$  with the boundary conditions  $y(2) = 1$  and  $y'(2) = -1$ . Find  $y(2.2)$  and plot  $y(x)$  with  $x$  between 2.0 and 2.3.

8. Solve exactly the simultaneous equations

$$\begin{aligned}4x + 5y &= 5 \\6x + 7y &= 7\end{aligned}$$

9. Using the Mathematica command `Solve`, find the solution of the equation

$$\sqrt{x+2} + 4 = x.$$

Note that `Solve` can solve this even though it is not a polynomial equation.

10. Compute the limit

$$\lim_{x \rightarrow \infty} \frac{1}{e^x - e^{x-x^2}}.$$

11. Find analytically the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = e^x \cos x, .$$

12. (a) Create a list of the integers from 0 to 10 and call it `xlist`.

(b) Create a list of the square integers from 0 to 10 and call it `ylist`.

(c) Generate the list whose  $i$ -th element is a two-element list  $\{\text{xlist}_i, \text{ylist}_i\}$ .

(d) Plot the points  $\{\text{xlist}_i, \text{ylist}_i\}$ .

*Hint:* For part (c) look at the Mathematica command `Transpose`.

13. Determine the first five positive roots of the Bessel function  $J_0(x)$  using the `FindRoot` command.

*Hint:* First graph the Bessel function to get an initial estimate of each of the roots.

14. The normalized eigenfunctions of the one-dimensional simple harmonic oscillator are

$$u_n(x) = 2^{-n/2} (n!)^{-1/2} \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} H_n(\sqrt{m\omega/\hbar} x) \exp\left(-\frac{m\omega x^2}{2\hbar}\right),$$

where  $H_n(t)$  is a Hermite polynomial. Work in units where the characteristic length scale,  $\sqrt{\hbar/(m\omega)}$ , is equal to unity.

(a) Show that

$$\int_{-\infty}^{\infty} u_3(x)^2 dx = 1,$$

which demonstrates that this wavefunction is normalized.

*Note:* In this question the integrals should be worked out *analytically*.

(b) Show that

$$\int_{-\infty}^{\infty} u_3(x)u_2(x) dx = 0,$$

which demonstrates that these two wavefunctions are orthogonal.

(c) Plot  $u_0, u_2$  and  $u_4$  on the same graph. Give a different style to each curve.

15. The current  $i$  in a driven electric circuit satisfies the equation

$$\frac{di}{dt} + 2i = \sin t,$$

with  $i(0) = 0$ .

(a) Determine  $i(t)$  numerically for  $t$  in the range 0 to 1.1.

(b) Determine  $i(t)$  analytically.

(c) Compare the numerical and analytical solutions for  $i(t)$  for  $t$  in the range between 0 and 1 in steps of 0.1.

*Note:* You should produce a neat table of results. Look up the command `TableForm` for to see how to do this.