PHYSICS 110A

Homework 9

Due in class, Tuesday, March 10. This is the last homework that will be graded. At a future lecture I will also give out one or two more questions (the material for which *may* be on the final) which will not be graded (for lack of time) but solutions will be available.

1. Two concentric metal spherical shells of radius a and b respectively, are separated by weakly conducting material of conductivity σ (see Griffiths Fig. 7.4(a)).
   (a) If they are maintained at a potential difference V, what current flows from one to the other?
   (b) What is the resistance between the shells?
   (c) Note that if $b \gg a$ the outer radius (b) is irrelevant. Exploit this observation to determine the current flowing between two metal spheres, each of radius a, immersed deep in the sea and held quite far apart (Griffiths Fig. 7.4(b)), if the potential between them if V.
   Note: This arrangement can be used to measure the conductivity of sea water.

2. A battery of emf $\mathcal{E}$ and internal resistance r is hooked up to variable "load" resistance R. If you want to deliver maximum possible power to the load, what resistance R should you choose (for fixed r and $\mathcal{E}$)?

3. We have said that a *time-independent* electric field in a conductor must be zero. However, suppose we perturb the conductor, for example by putting it in an *external* electric field. We will see in this question how fast the the charge will respond to "screen out" the electric field (i.e. make it zero again.)

   By using $\mathbf{J} = \sigma \mathbf{E}$ (σ is the conductivity), Gauss's law $\boldsymbol{\nabla} \cdot \mathbf{E} = \rho/\epsilon_0$, and the continuity equation $\boldsymbol{\nabla} \cdot \mathbf{J} + \partial\rho/\partial t = 0$, show that

   $$\frac{\partial \mathbf{E}}{\partial t} = -\text{const.}\, \mathbf{E},$$

   where you should determine the constant and hence deduce in what time the electric field decays.

   Determine the numerical value of this time for a conductor with $\sigma = 10^7$ (ohm-meters)$^{-1}$. *Note:* You should take not the precise numerical value seriously. The point is that the conductivity actually depends on frequency, and the dc value quoted here will be quite different from its value at the very high frequencies corresponding to the inverse of the time you found. Nonetheless, the main point of this question is still true: in a conductor **charges move very fast** to screen out an external field.

4. A metal bar of mass m slides frictionlessly on two parallel conducting rails a distance l apart (see Fig. 7.16 of Griffiths). A resistor R is connected across the rails and a uniform magnetic field $\mathbf{B}$ pointing into the page, fills the entire region.

(a) If the bar moves to the right at speed v, what is the current in the resistor? In what direction does it flow?

(b) What is the magnetic force on the bar? In what direction?

(c) If the bar starts off with speed v_0 at time $t = 0$, and is left to slide, what is its speed at a later time t?

(d) The initial kinetic energy of the bar was, of course, $\frac{1}{2}mv_0^2$. Check that the energy delivered to the resistor is exactly $\frac{1}{2}mv_0^2$.

5. A long solenoid, of radius a, is driven by an alternating current, so that the field inside is sinusoidal: $\mathbf{B}(t) = B_0 \cos(\omega t)\,\hat{\mathbf{z}}$. A circular loop of wire, of radius $a/2$ and resistance R, is placed inside the solenoid, and coaxial with it. Find the current induced in the loop, as a function of time.

6. A square loop, of side a, resistance R lies a distance s from an infinite wire that carries current I (see Griffiths Fig. 7.28). The wire is cut by scissors so the current drops to zero. Investigate this by assuming initially that current is turned down *gradually*:

$$I(t) = \begin{cases} (1 - t/\tau), & \text{for } 0 \le t \le \tau, \\ 0, & \text{for } t > \tau, \end{cases}$$

and then let $\tau \to 0$.
In what direction does the current in the loop flow? Find the total charge which passes a given point in the loop during the time which this current flows.

7. A small loop of wire (radius a) lies a distance z above the center of a large loop (radius b) (see Griffiths Fig. 7.36). The planes of the two loops are parallel, and perpendicular to the common axis.

(a) Suppose current I flows in the big loop. Find the flux through the little loop. (Assume that the little loop is so small that the field of the big loop is constant across the little loop.)

(b) Suppose current I flows in the little loop. Find the flux through the big loop. (Assume that the little loop is so small that you can treat it as a magnetic dipole.)

(c) Find the mutual inductances and confirm that $M_{12} = M_{21}$.

8. Find the self-inductance of the "hairpin" loop Griffiths Fig. 7.37.
(Neglect the contribution from the ends; most of the flux comes from the long straight sections.)
Note: You cannot ignore the thickness of the wire.

9. **Transformer**
Two coils are wrapped round a cylinder in such a way that the *same* flux passes through every turn of both coils (see Griffiths Fig. 7.54). The "primary" coil has N_1 turns per unit length, and the "secondary" coil has N_2 turns per unit length. Show that the emf in the secondary is given by

$$\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1},$$

where $\mathcal{E}_1$ is the (back) emf in the primary.
Note: The transformer delivers an output voltage V_2 (equal to $\mathcal{E}_2$ neglecting the resistance of the secondary coil) and receives an input voltage V_1 (equal to $\mathcal{E}_1$ neglecting the resistance of the primary). If $N_2 > N_1$ then the output voltage is greater than the primary. However, this does not violate conservation of energy because power is the product of the voltage and current and, if the voltage goes *up* the current goes *down* (see Griffiths Qu. 7.54 for details).