PHYSICS 110A

Homework 3

Due in class, Tuesday, January 27.

1. Find the potential inside and outside a uniformly charged solid sphere whose radius is R and whose total charge is q. Use infinity as your reference point.
   Hint: Use your result for $\mathbf{E}$ from Qu. 6 of Homework Set 2.

2. (a) Find the potential a distance s from an infinitely long straight wire that carries a uniform line charge λ.
   Note: You are *given* that the electric field is given by
   $$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{s}\,\hat{\mathbf{s}}\,.$$
   Hint: You can not set the reference point at ∞ because the charge itself extends to infinity. You need to set it at $s = a$ where a is arbitrary (except that it cannot be 0 or ∞).

   (b) Compute the gradient of V in each region and check that it yields the correct field.

3. Find the energy of a uniformly charged solid sphere of radius R and charge q. Do it in three ways:

   (a) Use Griffiths Eq. (2.43).
   Note: You found the potential in Qu. 1.

   (b) Use Griffiths Eq. (2.45).
   Note: Remember to integrate over all space. The electric field was determined in class. Either use your class notes or Gauss's theorem.

   (c) Use Griffiths Eq. (2.44). Take a sphere of large radius a.
   What happens for $r \to \infty$?

4. Two spherical shells, of radii a and b, are hollowed out of from the interior of a (electrically neutral) conducting sphere of radius R, see Griffiths Fig. 2.49. At the center of each cavity a point charge is placed—call these charges q_a and q_b.

   (a) Find the surface charges, σ_a, σ_b and σ_R.

   (b) What is the field outside the conductor?

   (c) What is the field in each cavity?

   (d) What is the force on q_a and q_b?

   (e) Which of these answers would change if a third charge, q_c, were brought near the conductor?

5. Find the capacitance per unit length of two coaxial metal cylinders of radii a and b, see Griffiths Fig. 2.53.

6. The energy of a parallel-plate capacitor is given by $W = \frac{1}{2}CV^2 = \frac{1}{2}Q^2/C$ where $C = A\epsilon_0/x$ in which x is the plate separation. Suppose the plate separation changes by a distance dx, keeping the charge on each plate constant. Compute the change in energy and hence the force on one plate due to the other.
   Ans: $F = -\frac{1}{2}\frac{Q^2}{\epsilon_0 A}$ where A is the area. The negative sign means that the force is attractive.

7. (a) Determine the general solution to Laplace's equation in spherical polar coordinates for the case where V only depends on r.

   (b) Do the same for cylindrical coordinates where V only depends on s (as defined by Griffiths).

8. Two semi-infinite grounded conducting plates meet at right angles. In the region between them there is a point charge q, see Griffiths Fig. 3.15.

   (a) Set up the image configuration and calculate the potential. You need to determine the charges and their locations.

   (b) What is the force on q?

   (c) How much work did it take to bring in q from infinity?

   (d) Suppose the planes met at some angle other than $90°$. Is it always possible eto solve the problem by the method of images? If not, for what particular angles *does* the method work?