1. By looking for a solution in the form a series find the general solution of the differential equation
\[(x^2 + 1)y'' - 2xy' + 2y = 0.\]

2. Consider the array of charges shown below. Determine the form of the electrostatic potential at large distance \( r \).

3. Use the recursion relation discussed in class to determine \( P_3(x) \) and \( P_4(x) \) by first expressing them terms of Legendre polynomials of lower order (whose form you may assume).

4. (a) Use the generating function to determine \( P_n(0) \) for \( n = 0, 1, \cdots, 6 \).
   (b) Find \( P_n(0) \) for general \( n \).
   
   *Note:* You will need to distinguish between even and odd \( n \).

5. Show that
\[ \int_{-1}^{1} x^m P_n(x) \, dx = 0 \quad \text{if} \quad n > m. \]

   *Hint:* Use the orthogonality property of the Legendre polynomials.

6. Obtain all the coefficients in a Legendre series for \( f(x) = x^4 \), i.e. writing
\[ f(x) = \sum_{n=0}^{\infty} a_n P_n(x) \]
for \( -1 \leq x \leq 1 \), determine the coefficients \( a_n \).

7. Find the first three non-vanishing terms in the Legendre series for the function \( f(x) \) sketched below.