A Rotated Ellipse

In this handout I have used *Mathematica* to do the plots. You may ignore the *Mathematica* commands and concentrate on the text and figures.

The equation of the ellipse we discussed in class is

\[ 9 x^2 - 4 xy + 6 y^2 = 5. \]

Here we plot it

\[
\text{ContourPlot}\left[ 9 x^2 - 4 xy + 6 y^2 = 5, \{x, -1, 1\}, \{y, -1, 1\}, \text{Axes} \to \text{True}, \text{Frame} \to \text{False}, \text{AxesLabel} \to \{"x", "y"\}, \text{AxesStyle} \to \text{Directive[Bold, Medium, Arrowheads[\{0, 0.05\}]}], \text{ContourStyle} \to \{\text{Red, Thick}\}, \text{PlotRange} \to \{(-1.1, 1.3), (-1.1, 1.3)\}\right]
\]

The major and minor axes of the ellipse are clearly rotated relative to the x and y axes. In class, we showed, by diagonalization, that the major axis lies along x' and the minor axis along y' where x' and y' are rotated relative to x and y by an angle \( \theta \) where \( \tan(\theta) = 2 \). In terms of the new axes, we showed that, the equation of the ellipse is

\[ x'^2 + 2 y'^2 = 1, \]

so the ellipse intersects the x' axis at x' = ±1 and the y' axis at y' = ±1/\( \sqrt{2} \). We plot the ellipse along with the rotated axes below.

\[
p1 = \text{ContourPlot}\left[ \{9 x^2 - 4 xy + 6 y^2 = 5, y = 2 x, y = - (1/2) x\}, \{x, -1, 1\}, \{y, -1, 1\}, \text{Frame} \to \text{False}, \text{Axes} \to \text{True}, \text{AxesStyle} \to \text{Directive[Bold, Medium, Arrowheads[\{0, 0.05\}]}], \text{ContourStyle} \to \{\text{Red, Thick}\}, \{\text{Blue, Thick}\}, \{\text{Blue, Thick}\}, \text{PlotRange} \to \{(-1.1, 1.3), (-1.1, 1.3)\}\right];
\]

\[
p2 = \text{Graphics}[\{\text{Blue, Thick, Arrowheads[\{0, 0.06\}]}, \text{Arrow}[\{\{-0.6, -1.2\}, \{0.6, 1.2\}\}], \text{Text}[\text{Style[x', Large, Blue]}, \{0.7, 1.1\}]\}];
\]

\[
p3 = \text{Graphics}[\{\text{Blue, Thick, Arrowheads[\{0, 0.06\}]}, \text{Arrow}[\{\{1, -1/2\}, \{-1, 1/2\}\}], \text{Text}[\text{Style[y', Large, Blue]}, \{-0.9, 0.58\}]\}];
\]
\[ \tan(\theta) = 2 \]

```mathematica
p4 = Graphics[{Text[Style[x, Large], {1.25, 0.1}], Text[Style[y, Large], {0.1, 1.25}]}];
p5 = Graphics[{Text[Style[\theta, Large, Orange], {0.35, 0.2}]}];
p6 = Graphics[{Text[Style["\tan(\theta) = 2", 18], {-0.65, 0.9}]}];
p7 = Graphics[{Orange, Thick, Arrowheads[{0, 0.06}], Arrow[BezierCurve[{{0.3, 0}, {0.3 Cos[0.3], 0.3 Sin[0.3]}, {0.3 Cos[0.6], 0.3 Sin[0.6]}, {0.3 Cos[1.1], 0.3 Sin[1.1]}}]}]}];
Show[p1, p2, p3, p4, p5, p6, p7]
```

`Rotated Ellipse.nb`