# Dynamical universality classes of the superconducting phase transition

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We present a finite temperature Monte Carlo study of the XY-model in the vortex representation, and study its dynamical critical behavior in two limits. The first neglects magnetic field fluctuations, corresponding to the absence of screening, which should be a good approximation in high $T_c$ superconductors ($\kappa \to \infty$) except extremely close to the critical point. Here, from finite size scaling of the linear resistivity we find the dynamical critical exponent of the vortex motion to be $z \approx 1.5$. The second limit includes magnetic field fluctuations in the strong screening limit ($\kappa \to 0$) corresponding to the true asymptotic inverted XY critical regime, where we find the unexpectedly large value $z \approx 2.7$. We compare these results, obtained from dissipative dynamics in the vortex representation, with the universality class of the corresponding model in the phase representation with propagating (spin wave) modes. We also discuss the effect of disorder and the relevance of our results for experiments.

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## I. INTRODUCTION

Considerable confusion currently exists, both theoretically and experimentally, regarding the dynamical universality class of the zero-field superconducting phase transition in high temperature superconductors. The short coherence length in these materials leads to large, non-gaussian fluctuations, and there is some experimental evidence that the static behavior is that of the 3-dimensional XY model[1–8], as in the lambda transition in superfluid helium. The dynamical universality class of the lambda transition in superfluid helium is that of a two component order parameter coupled to a conserved density which gives a propagating mode (second sound in $^4$He; a spin wave in the XY spin model) in the broken symmetry phase. This is model E in the notation of Hohenberg and Halperin.[9] However, the dynamical behavior of a superconductor could be very different, since the lattice acts as a momentum sink destroying Galilean invariance making the system more like helium in a porous medium. The existence of disorder does not destroy the Goldstone mode in the ordered phase but it does affect the vortices and the normal-fluid quasiparticles which tend to equilibrate with the lattice rather than co-moving with the condensate.

Furthermore, in d-wave superconductors, scalar disorder causes pair-breaking and quasiparticle branch recombination which may make it inappropriate to assume particle number conservation in the dynamics. There does not appear to have been any work to date on this question.

A related issue is that of the role of the long-range Coulomb interaction, which severely suppresses longitudinal current fluctuations, leaving only the transverse currents associated with vortices. However it is also possible that the low superfluid density and screening from the high normal fermion density will turn on the low energy longitudinal Carlson-Goldman fluctuations[10] of the order parameter as $T_c$ is approached. These microscopic fermionic effects may further confuse the data analysis and affect the width of the critical regime.

The issues raised above remain largely unresolved. The particular issues that we will explicitly discuss here are the role of magnetic screening and the role of disorder. In an extreme type-II, system, coupling to gauge fluctuations is weak,[1] but nonetheless, in principle, magnetic screening becomes important extremely close to the critical point where the system crosses over to inverted XY behavior.[11–13] The static correlations related to this issue have recently been studied numerically by Olsson and Teitel.[14] Here we will address the dynamics.

Most theoretical studies of critical dynamics in XY like spin systems have focussed on Landau-Ginsburg representations of the problem involving the *phase* (i.e. angle) of the spin. For static properties, there also exist equivalent dual representations[12,15–17] in terms of interacting *vortex* degrees of freedom. Although the static properties of the phase and vortex representations are the same, there is no reason, *a priori*, why the dynamical universality classes should be the same.

For the reasons discussed above it may be more appropriate, for superconductors, to consider a model with overdamped dissipative dynamics of the topological defects (the vortices). This is the approach that we will take here.

To add dynamics to the the phase representation, one can either include just dissipative dynamics, model A,[9] in which case the dynamical exponent, $z$, is close to 2, or one can incorporate the propagating (spin wave) modes, model E,[9] for which $z$ is exactly $3/2$ in three dimensions, ($d/2$ in $d$-dimensions). For the vortex representation, which has *discrete* variables, the natural dynamics is purely dissipative, such as that generated by Monte Carlo simulations (in which Monte Carlo time is equated with real time). Naively, it would seem unlikely that the *dissipative* dynamics of the vortex representation would be in the same universality class as the dynamics of model E (phase representation), which has *propagating* modes. Surprisingly, recent results by Weber and Jensen[18] come to the opposite conclusion. They find, for unscreened vortex interactions ($\kappa = \infty$) and overdamped dynamics, that the dynamical exponent is $z = d/2 = 1.5$, precisely the value one expects in model E dynamics, and considerably *less* than the value generally found with dissipative dynamics ($z \approx 2$).

In this paper we present results of Monte Carlo calculations of the dynamical critical exponents for the 3-dimensional XY model, in the vortex representation, with and without magnetic screening. For no screening, we confirm the unexpected result of Weber and Jensen[18], while by contrast, for strong magnetic screening, we find a rather large *enhancement* of $z$. Although it is known from the Harris criterion and verified numerically[19] that uncorrelated disorder is weakly irrelevant at the 3-dimensional XY critical point, the effect of such disorder on the dynamical properties is unknown.[20] We therefore also investigate the effects of disorder on the model with strong screening.

## II. THE MODELS

The model under investigation here is the XY–model with a fluctuating vector potential,

$$\mathcal{H} = -J \sum_{\langle i,j \rangle} \cos(\phi_i - \phi_j - \lambda_0^{-1} a_{ij}) + \frac{1}{2} \sum_{\square} [\nabla \times \mathbf{a}]^2, \quad (1)$$

where $J$ is the coupling constant (set to unity in the simulations), the $\phi_i$ denote the phases of the condensate on the sites $i$ of a simple cubic lattice of size $N = L^3$ with periodic boundary conditions. The sum is taken over all nearest neighbors $\langle i,j \rangle$. Additionally, we have a fluctuating gauge potential $a_{ij}$ with the gauge constraint $[\nabla \cdot \mathbf{a}] = 0$, and $\lambda_0$ denotes the bare screening length. The last term describes the magnetic energy, where the sum runs over all elementary plaquettes on the lattice and the curl is the directed sum of the gauge potential round a plaquette.

In our work, the main focus is equilibrium vortex dynamics. However, vortices are only *implicitly* present in the above model through the relation

$$\oint \nabla\phi(\mathbf{r}) \cdot \mathrm{d}\mathbf{r} = 2\pi n, \quad (2)$$

where $n = 0, \pm 1, \pm 2, ...$ denotes the net vorticity encircled by the integration contour. In order to analyze the critical behavior of this model due to vortex fluctuations *explicitly*, it is easier to go from the above *phase* representation of the XY–model to its *vortex* representation. This is achieved by replacing the cosine in Eq. (1) with the periodic Villain function and performing fairly standard manipulations[12,15–17] to obtain

$$\mathcal{H}_V = \frac{1}{2} \sum_{i,j} \mathbf{n}_i \cdot \mathbf{n}_j \, G_{ij}[\lambda_0]. \quad (3)$$

Here, the $\mathbf{n}_i$ are vortex variables which sit on the links of the *dual lattice* (which is also a simple cubic lattice) and $G_{ij}$ is the screened lattice Green's function

$$G_{ij}[\lambda_0] = J \frac{(2\pi)^2}{L^3} \sum_{\mathbf{k}} \frac{\exp[\mathrm{i}\,\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)]}{2 \sum_{m=1}^{3} [1 - \cos(k_m)] + \lambda_0^{-2}}. \quad (4)$$

In the long range case, $\lambda_0 \to \infty$, the divergent $\mathbf{k} = \mathbf{0}$ contribution has to be excluded from the sum and the constraint $\sum_i \mathbf{n}_i = \mathbf{0}$ has to be imposed. In the short range case (finite $\lambda_0$) this is not necessary. The transformations involved in going from the phase to the vortex representation also yield the local constraints $[\nabla \cdot \mathbf{n}]_i = 0$, i.e., there are no magnetic monopoles (zero divergence constraint). It is important to note, that the vortex representation does not contain spin wave degrees of freedom anymore, since they have been "integrated out" in the transformation procedure.

We will be interested in two limits of the above model: (i) no screening ($\lambda_0 \to \infty$), corresponding to the extreme type–II limit ($\kappa \to \infty$), where the individual vortex lines have long range interactions; (ii) strong screening ($\lambda_0 \to 0$), i.e., short range interactions, which is supposed to be the correct description of a superconductor extremely close to the critical point,[1] though the size of the critical region where such screening is relevant may be too small to be observable in practice. In this limit the interaction reduces to $G_{ii} = J(2\pi\lambda_0)^2$ and $G_{i \neq j} = 0$ (plus exponentially small corrections of order $\exp(-r/\lambda_0)$). We will in this case use units where $J(2\pi\lambda_0)^2 = 1$. The resulting Hamiltonian is of the very simple form

$$\mathcal{H}_V = \frac{1}{2} \sum_{i} \mathbf{n}_i \cdot \mathbf{n}_i, \quad (\lambda_0 \to 0). \quad (5)$$

Note, however, that this Hamiltonian is not trivial, since the constraints on the local divergence effectively generate interactions among the $\mathbf{n}_i$. Note further, that this is also the dual representation of the XY model *without* screening, in which the temperature scale is inverted.[12,17,21] The static universality class of Eq. (5) is then the same as that of Eq. (3) with $\lambda_0 = \infty$, and is given by XY exponents.[22] The dynamical universality class, however, may be different and determining it is one of the goals of our study.

In addition to the pure short-range model, we also study the short-range model with quenched random local $T_c$. In this case we replace Eq. (5) by

$$\mathcal{H}_V = \frac{1}{2}\sum_{i,\mu} \xi_{i\mu} n_{i\mu}^2, \quad (\lambda_0 \to 0,\ \xi_{i\mu} \text{ random}). \tag{6}$$

where $\xi_{i\mu}$ is uniformly distributed in the interval $[0.5, 1.5]$.

## III. MONTE CARLO SIMULATION AND FINITE SIZE SCALING

We simulate the following model Hamiltonians in the vortex representation:

1. Eq. (3) with $\lambda_0 \to \infty$
2. Eqs. (5) and (6), which corresponds to $\lambda_0 \to 0$.

We take simple cubic lattices of size $L^3$ where $4 \leq L \leq$12–64. Periodic boundary conditions are imposed. We start with configurations with all $\mathbf{n}_i = \mathbf{0}$, which clearly satisfies the constraints, and a Monte Carlo (MC) move consists of trying to create a closed vortex loop around a plaquette.[23] This trial state is accepted according to the heat bath algorithm with probability $1/[1+\exp(\beta\Delta E)]$, where $\Delta E$ is the change of energy and $\beta = 1/T$.

Each time a loop is formed it generates a voltage pulse $\Delta Q = \pm 1$ perpendicular to its plane, the sign depending on the orientation of the loop. This leads to a net electric field[24]

$$E(t) = \frac{h}{2e} J^V(t) \quad \text{with} \quad J^V(t) = \frac{\Delta Q}{\Delta t}, \tag{7}$$

where $J^V(t)$ is the vortex current density, and $\Delta t = 1$ for one full sweep through the system, where, on average, an attempt is made to create or destroy one vortex loop per plaquette.

The nonlinear I–V characteristics of the inverted XY model can be modeled as the electric field $E$, due to vortex current response in the presence of a uniform Lorentz force on the vortex lines, proportional to the applied current density $J$.[25] In addition, the linear response resistance can be calculated from the equilibrium voltage–voltage fluctuations via the Kubo formula[26]

$$R = \frac{1}{2T} \sum_{t=-\infty}^{\infty} \Delta t \langle V(t) V(0) \rangle \tag{8}$$

Here, $\langle \cdots \rangle$ denotes the thermal average, and the voltage across the sample is $V(t) = LE(t)$. The resistivity is $\rho = L^{d-2} R$.

Since we are working with lattices of finite length $L$, one has to employ finite size scaling techniques to extract the critical behavior. A detailed scaling theory has been developed for superconductors by Fisher et al.[1] and we now summarize the results from it which will be needed for our data analysis.

Near a second order phase transition the linear resistivity obeys the scaling law

$$\rho_{\text{lin}}(T, L) = L^{-(2-d+z)}\, \tilde{\rho}\left[L^{1/\nu}(T - T_c)\right], \tag{9}$$

where $\nu$ is the correlation length exponent, $z$ is the dynamical exponent and $\tilde{\rho}$ is a scaling function. At the critical temperature, $\tilde{\rho}(0)$ becomes a constant and therefore $\rho_{\text{lin}}(T_c, L) \sim L^{-(2-d+z)}$. If we plot the ratio of $\rho_{\text{lin}}$ for different system sizes against $T$, then

$$\frac{\ln[\rho_{\text{lin}}(L)/\rho_{\text{lin}}(L')]}{\ln[L/L']} = d - 2 - z \quad \text{at } T_c, \tag{10}$$

i.e., all curves for different pairs $(L, L')$ should intersect and one can read off the values of $T_c$ and $z$. We will refer to this kind of data plot as the *intersection method.* With the values of $T_c$ and $z$ determined by the intersection method we can then use a scaling plot according to Eq. (9) to obtain the value of $\nu$.

A similar analysis[1,20] shows that, above a characteristic current scale $J_{\text{nl}}$, which varies as $\sim L^{-2}$ at the critical point, non-linear response sets in and the electric field varies as

$$E \sim J^{(1+z)/2}. \tag{11}$$

It is useful to locate the critical temperature from equilibrium properties instead of the dynamic scaling of the linear resistivity, since such simulations are easier to converge. In the short-range case we do this by considering an ensemble with a fluctuating winding number $W$, defined by

$$W_\mu = \frac{1}{L}\sum_i n_{i\mu}. \tag{12}$$

From the finite size scaling relation

$$\langle W_\mu^2 \rangle = f(L^{1/\nu}(T - T_c)) \tag{13}$$

we can locate the critical point in the short range case, both with and without disorder using the fact that the winding number is scale invariant at the critical point. Note that the simulations of this quantity require global moves, where vortex lines going all the way through the system are created and destroyed, and therefore do not represent the dynamics of the system in a realistic way. This does not pose a problem since we use this calculation solely as a way to accurately locate the critical point and not to follow the dynamics.

Because different winding number classes are difficult to equilibrate efficiently in large systems in this model, we use an (exact) duality transformation from the short-range vortex models, Eqs. (5) and (6), back to an XY phase model with a Villain interaction:[15,12]

$$\begin{aligned} Z_V &= \sum_{\{\mathbf{n}_{i\mu}\}} \delta_{\nabla\cdot\mathbf{n}_i,0}\, e^{-\frac{1}{2T}\sum_{i\mu}\xi_{i\mu}\mathbf{n}_{i\mu}^2} \\ &= \int_0^{2\pi} \left[\prod_i d\theta_i\right] \sum_{\{m_{i\mu}\}} e^{-\sum_{i\mu}\frac{T}{2\xi_{i\mu}}\left(\theta_i - \theta_{i+\mathbf{e}_\mu} - 2\pi m_{i\mu}\right)^2} \end{aligned} \tag{14}$$

(a constant prefactor was suppressed in the last equality). Performing the sum over the integer dummy variables $m_{i\mu}$ leaves a phase-only model. This phase representation allows us to take advantage of the Wolff algorithm in the simulation which largely circumvents problems of critical slowing down and equilibration in the limit of large system sizes.[27] The spin-wave stiffness of the dual model is given by $\rho_s = \partial^2 f / \partial \tilde{A}_\mu^2 |_{\tilde{A}=0}$, where $\tilde{A}_\mu$ is a constant vector potential added to the phase gradient in Eq. (14). This is related to the winding number fluctuations in the vortex model by $\rho_s = T L^{2-d} \langle W_\mu^2 \rangle$.

## IV. DATA ANALYSIS

In this section we analyze our simulation data for the XY-model in the vortex representation, starting with the model in Eq. (3) with long range interactions, i.e., neglecting screening ($\lambda_0 \to \infty$).

### A. Long range interactions

In Fig. (1) we show the data for the linear resistivity $\rho_{\rm lin}$ plotted vs. $T$ obtained from a simulation of the Hamiltonian (3) with $\lambda_0 = \infty$. One observes that at high temperatures ($T = 3.1$) there is hardly any size dependence in the data, consistent with a correlation length which is smaller than the system size. As one goes to lower temperature, the size dependence becomes stronger. This indicates critical behavior, since in the limit $L \to \infty$ the linear resistivity should go to zero below $T_c$.

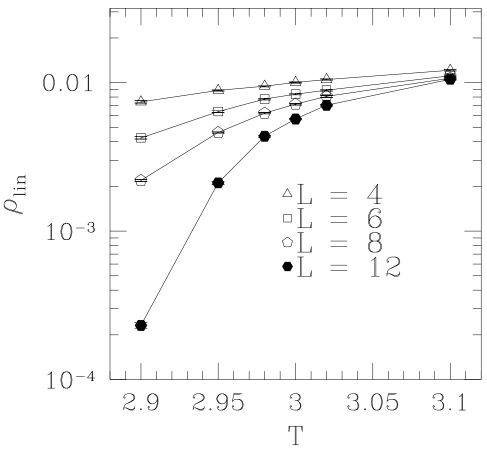


FIG. 1. Linear resistivity $\rho_{\rm lin}$ vs. $T$ for the vortex model of Eq. (3) without screening, i.e. $\lambda_0 = \infty$.

However, since it is impossible to locate the critical temperature by this kind of plot, we show, in Fig. (2), the same data of Fig. (1) plotted according to the intersection method. The curves intersect at approximately $T_c = 3.01\,(\pm 0.01)$ and $y \approx -0.45$ corresponding to $z = 1.45\,(\pm 0.05)$, very similar to the result of Weber and Jensen, who find $z = 1.51\,(\pm 0.03)$.[18] Also, the value of $T_c$ agrees nicely with the one obtained earlier from simulations by Dasgupta and Halperin.[12]

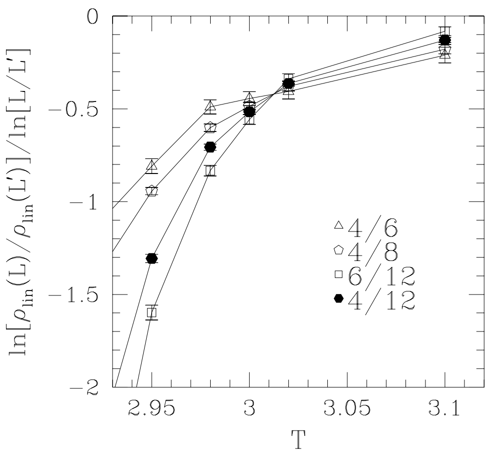


FIG. 2. Linear resistivity, plotted according to the *intersection method*, vs. $T$ with the data of Fig. (1). At the intersection one can read off $T_c \approx 3.01$ and $y \approx -0.45$, corresponding to $z \approx 1.45(\pm 0.05)$.

Having established $T_c$ and $z$ we can now perform a scaling plot of the data according to Eq. (9). In Fig. (3) we plot $\rho_{\rm lin} L^{2-d+z}$ vs. $L^{1/\nu}(T - T_c)$ and find that the data collapses best with $T_c = 3.01 \pm 0.01$, $z = 1.5 \pm 0.05$ and $\nu = 0.66 \pm 0.01$. This independent result confirms that the dynamical scaling ansatz for $\rho_{\rm lin}$ yields the expected value of the correlation length exponent $\nu$ as well as a consistent value for $z$.

### B. Short-Range Interactions

In this section we will first describe how we locate the critical point of the pure and disordered short range models defined by Eqs. (5) and (6), using finite size scaling analysis of Monte Carlo data for static quantities. Then, using dynamic scaling analysis we determine the dynamical exponents both from equilibrium vortex dynamics simulations, and from driven nonequilibrium simulations.

In our simulations of Eq. (5), we used $10^6 - 10^7$ sweeps to calculate averages, and discard the initial 10% of the data for equilibration. For the disordered case we average over $10 - 100$ samples to obtain small fluctuations.

As noted earlier, for the purpose of locating the critical point as precisely as possible for both the pure and random short-range models, we found it convenient to perform an exact transformation from the inverted XY model back to the phase representation. We compute the

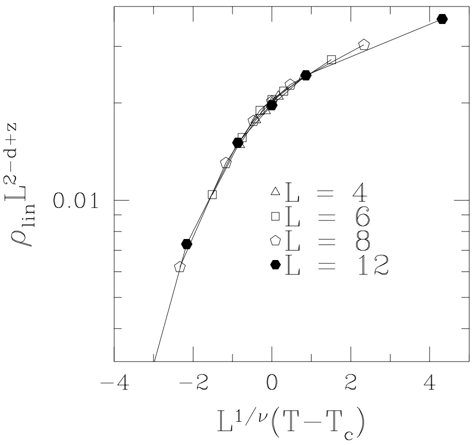


FIG. 3. Scaling plot of $\rho_{\rm lin}$ according to Eq. (9). Best scaling was achieved with $T_c = 3.01\,(\pm 0.01)$, $\nu = 0.66\,(\pm 0.01)$, and $z = 1.5\,(\pm 0.05)$.

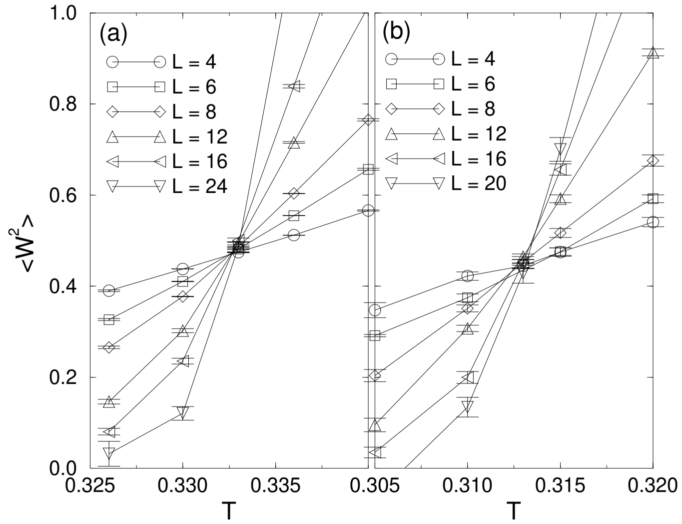


FIG. 4. Plot of Monte Carlo data for the the winding number fluctuations, $\langle W^2\rangle$ vs. temperature $T$ for (a) pure and (b) dirty 3D screened vortex models. According to scaling theory (see text) the critical temperature is where curves (formed by straight-line interpolation between data points) intersect.

spin stiffness $\rho_s$ using the Wolff algorithm to overcome the critical slowing down and the difficult problem of equilibrating different winding number classes.[27,19] From hyperscaling, $\langle W^2\rangle = L^{d-2}\rho_s/T$ is scale invariant at the critical point.[28] Thus curves for different $L$ all cross at $T = T_c$ as shown in Fig. 4. Using this method, we find for the pure model $T_c = 0.333 \pm 0.001$ (which agrees nicely with the *inverse* of $T_c$ for the long range model) and $T_c = 0.313 \pm 0.001$ for the disordered model. Of course, this technique cannot be used to speed up the determination of $z$ since the Wolff algorithm intentionally does not represent local relaxation dynamics. Note also that the values of $\langle W^2\rangle$ at the crossing points in Fig. 4 in the pure (a) and disordered (b) cases agree within the error bars, which is what we expect from two-scale factor universality[29] and the fact that disorder is weakly irrelevant to the statics.

Simulations were then carried out at the measured critical temperatures to compute the dynamics in the vortex representations. In Fig. (5) the dynamical exponent $z$ of the 3D loop model is determined from Monte Carlo data for the linear resistivity computed from the Kubo formula, Eq. (8).[26] Using $\rho_{\rm lin} \sim L^{1-z}$ at $T = T_c$, and from a power law fit to the data we get $z = 2.7\pm 0.1$. This value is consistent with, but more accurate than, the result of Wengel and Young[21]. We also verified that is possible to collapse the data for different sizes and temperatures using Eq. (9) with $\nu \approx 2/3$.

In the case of the disordered vortex model, it is convenient to determine $z$ from the nonlinear I-V characteristics instead of the linear resistivity. Driving the system out of equilibrium makes it cheaper to converge the simulations, which is desirable for the disorder averaging. Fig. (6) shows data for the nonlinear I-V characteristics. For the largest size studied, $L = 64$, it is reasonable to assume that the data is in the range where $J > J_{\rm nl} \sim 1/L^2$. Hence, according to Eq. (11), the dynamical exponents can be obtained from power law fits of the form $E \sim J^{(1+z)/2}$ to the data at $T_c$. The fits were done in the current interval where a power law best describes the data, leaving out the highest currents where saturation artifacts in the simulation limit the voltage. This gives $z_{\rm pure} = 2.60 \pm 0.1$ and $z_{\rm dirty} = 2.69 \pm 0.1$. The small discrepancy between these values is within the statistical uncertainty of the simulations, and the values are consistent with the exponent obtained from the calculation of the linear resistivity. The coincidence of $z$ for the pure model with the result of linear resistivity gives a consistency test showing that the nonlinear I-V characteristics correctly determines $z$. In particular, our model assumption of a uniform current driving the vortices in the presence of the disorder does not seem to introduce any errors.

## V. SUMMARY AND CONCLUSION

We have performed simulations of the dynamics of the 3-dimensional XY model in a vortex representation with and without magnetic screening. Without screening, we find $z \approx 3/2$, in agreement with earlier work of Weber and Jensen[18], who note that this result agrees with the dynamical critical exponent of the phase model with spin wave degrees of freedom, model E[9]. However, the spin wave degrees of freedom are *separated* from the vortex degrees of freedom when going to the vortex representation[15–17] and the remaining vortex degrees of freedom have only dissipative motion. Hence we find it quite surprising, that the exponents from spin wave and vortex dynamics agree numerically. We

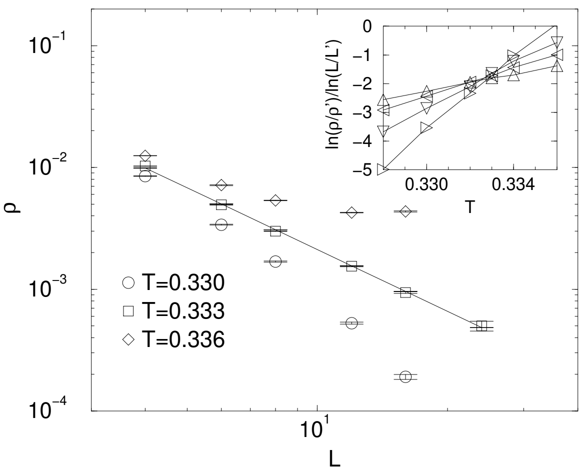


FIG. 5. Determination of the dynamical critical exponent from MC data for the linear resistivity $\rho$ computed using the Kubo formula for the pure screened vortex model. A power law fit to the data (straight line) for $T = T_c$ gives the exponent $z = 2.7 \pm 0.1$. Inset: Data plotted using the intersection method similarly as in Fig. 2. The curves have $L/L' = 4/6$ (triangles up), 6/8 (triangles left), 8/12 (triangles down), and 12/16 (triangles right) and give the same values for $z$.

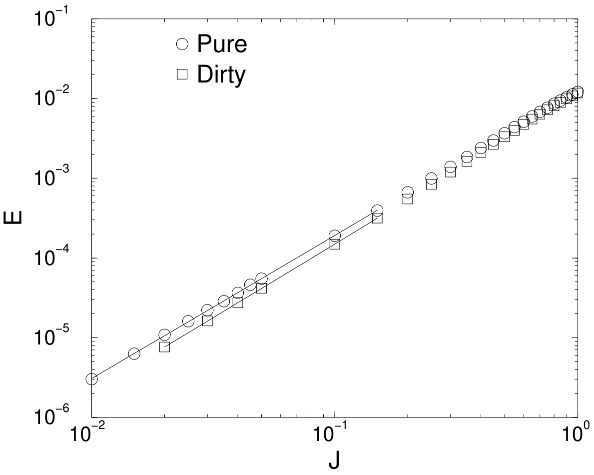


FIG. 6. Determination of the dynamical critical exponent from MC data for the nonlinear I-V characteristics for system size $64^3$. Power law fits (solid lines) in the interval of currents where the data is best described by a power law, with slope $(1+z)/2$ determines $z$. The dynamical exponents in both cases agree (within the statistical uncertainty) with the value $z \approx 2.7$ found in Fig. 5.

note that there is some experimental evidence for super-diffusive 3-dimensional XY dynamics in superconductors with $z \sim 1.25 - 1.5$, but only for the case of finite magnetic fields.[8,6]

One can try to argue that it is the long-range forces among the vortices that accounts for the super-diffusive (i.e. $z < 2$) behavior. Indeed one can even argue that the spin waves are implicitly present since they are what mediate the long-range forces. However, we note that, in this model, the long-range forces are *instantaneous* and not retarded. Hence it is still a mystery to us why the vortex model appears to be consistent numerically with Model E dynamics. Note that Lee and Stroud,[30] who measured $I - V$ characteristics on the resistively shunted junction model (which is described by Model E dynamics) using Langevin (as opposed to Monte Carlo) dynamics, find $z = 1.5\,(\pm 0.5)$, as expected for this model.

The value of $z$ for the short-range case does not seem to be significantly affected by the disorder, and so disorder appears to be irrelevant dynamically as well as statically. In both the pure and dirty case however, $z$ is significantly larger than is usually seen in relaxational dynamics where $z$ is typically only slightly larger than two.[9] There is some experimental evidence for such enhanced values of $z$. Anlage's group[4] finds $\nu = 1.0 \pm 0.2$, and $z = 2.65 \pm 0.3$. The value of $z$ is measured directly at the critical point from the resistivity scaling and is probably more reliable than the value of $\nu$ which is measured somewhat more indirectly (necessarily) using data away from the critical point.[31] It is possible therefore that the value of $\nu$ is actually consistent with the 3DXY value of 0.667. Moloni et al.[7] find $z = 2.3 \pm 0.2$.

These experimental values are consistent with the value we obtain here. However, in the extreme type-II limit, the inverted XY critical regime is expected to be very narrow and difficult to access experimentally. It is therefore unclear at this point how significant this agreement is. Further work is needed to estimate more precisely the crossover point to inverted XY behavior in real materials.

To conclude, we have raised a number of issues about experimental and theoretical uncertainties regarding the dynamical universality class of the superconducting phase transition in high $T_c$ superconductors. Clearly considerably more work needs to be done to address this problem. One aspect that we have not yet addressed is the effect of disorder in the case of long-ranged unscreened interactions. A second problem worth pursuing is the precise theoretical relationship between the various possible spin wave dynamics in the phase representation and the corresponding dynamics of the same model in the various dual (vortex) representations.

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