Finite Temperature Ordering in the Three-Dimensional Gauge Glass

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We present results of Monte Carlo simulations of the gauge glass model in three dimensions using exchange Monte Carlo. We show for the first time clear evidence of the vortex glass ordered phase at finite temperature. Using finite size scaling we obtain estimates for the correlation length exponent, $\nu = 1.39 \pm 0.20$, the correlation function exponent, $\eta = -0.47 \pm 0.07$, and the dynamic exponent $z = 4.2 \pm 0.6$. Using our values for z and ν we calculate the resistivity exponent to be $s = 4.5 \pm 1.1$. Finally, we provide a plausible lower bound on the the zero-temperature stiffness exponent, $\theta \geq 0.18$.

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I. INTRODUCTION

High-temperature superconductors have a phase diagram that is rich with physically diverse phenomena[1]. In the mixed state of a pure type-II system one finds the Abrikosov lattice[2] of triangularly arranged vortices. This vortex lattice prohibits superconductivity because any perpendicularly applied current produces a Lorentz force causing the vortices to move, dissipating energy. The addition of disorder, however, drastically changes the behavior of the mixed state. Correlated disorder, such as from heavy ion irradiation[3] or twin boundaries,[4] causes the vortex lines to locally align with and adhere to the defects; this destroys the long-range order of the lattice and produces a superconducting glassy phase known as the Bose glass[5]. Random point disorder, e.g., from proton irradiation[6], pins the vortices in random positions creating a different type of superconducting phase known as the vortex glass[7,8].

Similar to a spin glass, the ordered state of the vortex glass is characterized by the phase of the superconducting order parameter randomly oriented in space but frozen in time. A system must have a Hamiltonian that contains both randomness and frustration in order to exhibit this type of behavior. One such model that has been used extensively to simulate the vortex glass transition is the gauge glass[1,9].

The gauge glass is important to study because like other spin glasses in 3 spatial dimensions,[10–12] the existence of a finite temperature transition within the gauge glass is controversial. The first estimate of the critical temperature was given in 1990 by Huse and Seung[9], $0 < T_c \leq 0.6$. A year later this estimate was improved significantly by Reger et al.,[13] who found that $T_c = 0.45 \pm 0.05$, but their data was insufficient to establish that spin glass order occurs below this temperature. Subsequent studies[14–19] have focused on defect energy scaling to determine the existence of a finite T_c by calculating the stiffness exponent θ. Unfortunately, these results seem to be dependent on the chosen boundary conditions, and there is little agreement on the values of θ. Consequently it is not clear whether the lower critical dimension is 3 or less than 3.

In this paper we attempt to settle the controversy by studying the gauge glass using exchange Monte Carlo,[20,21] also known as parallel tempering.[22–24] This technique allows us to simulate lower temperatures and larger sizes than previously possible. The main features of our work are as follows.

1. We present for the first time clear evidence of vortex-glass ordering at finite temperature in the three-dimensional gauge glass model. We present superior data to that previously available[13] and obtain a more accurate determination of the critical temperature $T_c/J = 0.47 \pm 0.03$.

2. Using finite size scaling we determine the correlation length exponent $\nu = 1.39 \pm 0.20$ to a higher degree of precision then earlier works.[30,13] The correlation function exponent is also obtained, $\eta = -0.47 \pm 0.07$. To our knowledge, this is the first numerical estimate of η for the gauge glass from simulations. Assuming hyperscaling, these two exponents completely determine the universality class of the gauge glass model.

3. Using standard Monte Carlo, we determine the dynamical exponent $z = 4.2 \pm 0.6$ and compare our results to experimental measurements of the resistivity exponent $s = \nu(z - 1)$. We find $s = 4.5 \pm 1.1$.

The layout of the paper is as follows. In §II we describe the model while in §III we discuss the observables that we measure. In §IV we discuss our implementation of the exchange Monte Carlo method and our tests for equilibration. Our results for statics are discussed in §V while our results for dynamics are given in §VI. In §VII we summarize our results and give some perspectives for future work.

II. THE MODEL

The gauge glass describes the physics of a disordered type-II superconductor at distances larger than some characteristic length scale Λ, beyond which order in the flux lattice has been broken.[1,25] One can then imagine the system as a granular superconductor (in an applied magnetic field) with an inter-grain separation of order Λ. Such a system can be modeled as an array of nearest neighbor coupled Josephson junctions.[9,26] This leads to the Hamiltonian

$$\mathcal{H} = -J \sum_{\langle i,j \rangle} \cos(\phi_i - \phi_j - A_{ij}), \tag{1}$$

where each site i on an $N = L \times L \times L$ cubic lattice has an associated phase angle ϕ_i, J is a positive ferromagnetic (Josephson) coupling between nearest neighbors, and A_{ij} is proportional to the line integral of the vector potential along a straight line path from site i to its nearest neighbor site j,

$$A_{ij} = \frac{2\pi}{\Phi_0} \int_{\vec{r}_i}^{\vec{r}_j} \vec{A} \cdot d\vec{l}. \tag{2}$$

$\Phi_0 = hc/(2e)$ is the flux quantum.

The Hamiltonian of the gauge glass is given by Eq. (1) in which the A_{ij} *are quenched random variables* uniformly distributed from 0 to 2π. Note that, by contrast, restricting the A_{ij} only to values 0 and π leads to the XY spin glass, which is equivalent to setting $A_{ij} = 0$ and taking the interactions to be $\pm J$ at random.

The gauge glass is perhaps the simplest model of a disordered type-II superconductor that contains the correct order parameter symmetry as well as the randomness and frustration necessary for glassy behavior. However, there are some features it ignores.[9]

The model ignores screening since the A_{ij} in Eq. (1) are quenched; there are no thermal fluctuations in the magnetic field. This corresponds to the extreme type-II limit in which the Ginzburg-Landau[27] parameter $\kappa = \lambda/\xi \to \infty$, where λ is the penetration depth and ξ is the coherence length. This limit may be realistic since κ can be quite large in high T_c superconductors, e.g., $\kappa \approx 90$ for $YBa_2Cu_3O_{7-\delta}$. It seems, however, that when the gauge glass is modified to include strong screening, the finite temperature transition to the vortex glass phase is rounded out in three dimensions very close to the putative T_c.[28–33] This rounding probably takes place over such a narrow temperature region that will be very difficult to observe. Hence a model which neglects screening, like the gauge glass, should be able to account for most of the observable data in the critical region.

Unlike a real superconductor in a magnetic field, the gauge glass is isotropic on average. There are local quenched fluxes but no net field in any direction. In six or more spatial dimensions the lack of anisotropy does not seem to matter[34]; however it is still an open question whether, in three dimensions, the vortex glass transition in the gauge glass and in a system with a non-zero net field are in the same universality class.

The source of the quenched randomness in the gauge glass model is the vector potentials linking the sites. This is not very realistic, and a more accurate model would have vector potentials corresponding to a uniform field, and put disorder into the strength of the couplings. However, the precise details of the disorder should be irrelevant for critical phenomena

In this paper we shall show very clearly that the gauge glass has a finite temperature transition to a vortex glass ordered state and that its critical exponents agree, within fairly large error bars, with some experimental measurements on type-II superconductors. More work remains to be done to check whether an anisotropic model with a net field would change the universality class.

III. THE OBSERVABLES

A standard technique to determine the critical temperature is to use the Binder ratio[35]

$$g(L) = 2 - \frac{[\langle |q|^4 \rangle]_{\rm av}}{[\langle |q|^2 \rangle]^2_{\rm av}}, \tag{3}$$

where $[\cdots]_{\rm av}$ denotes an average over configurations of the disorder, $\langle \cdots \rangle$ denotes a thermal average and q is the complex overlap order parameter

$$q = \frac{1}{N} \sum_{j=1}^{N} \exp[i(\phi_j^\alpha - \phi_j^\beta)], \tag{4}$$

in which α and β denote two independent replicas with the same disorder. One plots $g(L)$ vs. T for different L and identifies the temperature at which the curves cross as T_c. However, since $g(L)$ cannot exceed unity, the splaying out of the data for $g(L)$ below T_c (which indicates spin glass order) is a small effect which can be difficult to see. This is why it as been so hard to establish conclusively that there is spin glass order at finite-T in the three-dimensional Ising spin-glass.[10–12]

In order to avoid this problem we follow Reger et al.[13] in calculating a current; this is defined as the rate of change of the free energy with respect to a twist angle Θ at the boundaries. We begin by replacing periodic boundary conditions with twisted boundary conditions along one of the axes $\hat{x}$, i.e.

$$\phi_{i+L\hat{x}} = \phi_i + \Theta. \tag{5}$$

Note that $\Theta = 0$ corresponds to periodic boundary conditions, and $\Theta = \pi$ corresponds to anti-periodic boundary conditions. We can convert the model back to periodic boundary conditions by redefining ϕ through

$$\phi_{i+n\hat{x}} \to \phi_{i+n\hat{x}} - \frac{n}{L}\Theta. \tag{6}$$

The model then is precisely Eq. (1) with periodic boundary conditions but with the A_{ij} for bonds in the x-direction changed according to

$$A_{i,i+\hat{x}} \to A_{i,i+\hat{x}} - \frac{\Theta}{L}. \quad (7)$$

Using $F = -\ln(Z)/\beta$ we define a current as the response of the free energy to an infinitesimal Θ,

$$I(L) \equiv \lim_{\Theta\to 0} \frac{\partial F}{\partial \Theta} = \frac{1}{L}\sum_i \langle \sin(\phi_i - \phi_{i+\hat{x}} - A_{i,i+\hat{x}})\rangle. \quad (8)$$

Because the A_{ij} are uniformly distributed over the entire period of the sine function, the the value of the current averaged over samples is zero, i.e.

$$[I(L)]_{\rm av} = 0. \quad (9)$$

Consequently we calculate the root-mean-square current, given by

$$I_{\rm rms} \equiv \sqrt{[\langle I_\alpha(L)\rangle\langle I_\beta(L)\rangle]_{\rm av}}, \quad (10)$$

where α and β denote two independent replicas with the same disorder. We use two replicas to avoid any bias in calculating the average of the square of the current.

The primary advantage of using $I_{\rm rms}$ rather than $g(L)$ is that $I_{\rm rms}$ increases with L for $T < T_c$, so the splaying out the data below T_c should be much easier to see than for the Binder ratio. In the ordered phase the current should scale with the stiffness exponent[36,14–19] θ, i.e.

$$I_{\rm rms} \sim L^\theta, \qquad (T < T_c), \quad (11)$$

where $\theta > 0$ if $T_c > 0$. Above T_c, where spin-spin correlations are small, we expect $I_{\rm rms}$ to approach zero with increasing L because larger systems are less sensitive to a twist at the boundaries. The $I_{\rm rms}$ curves cross at T_c and have the finite size scaling form

$$I_{\rm rms} = \tilde{I}(L^{1/\nu}t) \quad (12)$$

where

$$t = (T - T_c)/T_c \quad (13)$$

is the reduced temperature.

IV. EXCHANGE MONTE CARLO AND EQUILIBRATION

Exchange Monte Carlo,[20,21] also called parallel tempering,[22–24] is a technique for simultaneously simulating multiple copies of a particular configuration of disorder with each copy at a different temperature. After a certain number of sweeps through the lattice, one tries to exchange the spin configurations of copies at neighboring temperatures with the probability

$$P(\sigma_m \leftrightarrow \sigma_{m+1}; \beta_m, \beta_{m+1}) = \exp(-\Delta) \quad (14)$$

where

$$\Delta \equiv (\beta_m - \beta_{m+1})(E(\sigma_{m+1}) - E(\sigma_m)). \quad (15)$$

σ_m is the spin configuration of the mth copy which has inverse temperature β_m and total energy $E(\sigma_m)$. A given spin configuration is thus heated and cooled many times during the simulation. Since equilibration is fast at high temperature, each time the system is cooled the minimum (valley) that it visits is uncorrelated with the minimum that it visited at the previous cooling. Hence the system can can visit different local minima at low temperature more efficiently than if the temperature were kept fixed. In the latter case, very large barriers would have to be overcome, which takes a time *exponentially* large in the ratio of the barrier height to the temperature.

In deciding what temperatures to simulate, one would like the energy distributions of neighboring temperatures to have enough overlap that the probability to exchange configurations is sufficiently high. This requires

$$\frac{T_{m+1} - T_m}{T_m} = c_m N^{-1/2}, \quad (16)$$

where the c_m are constants of order unity. In our simulations we took all the c_m to be exactly unity, which gave a satisfactory acceptance ratio for temperature exchanges in the interval from 0.5 to 0.6 for all sizes and temperatures studied.

We took the highest temperature, $T_{\rm max}$, to be approximately $2T_c$ at which the spin dynamics are quite rapid. Temperature exchanges were carried out after every 10 Monte Carlo sweeps, after which "time" the normalized energy-energy auto-correlation function at $T_{\rm max}$ is quite low, about 0.25.

The equilibration time, $t_{\rm eq}$, for $I_{\rm rms}$ is determined by following the temporal evolution of

$$I^2(t_0) \equiv \left[\frac{1}{t_0}\sum_{t=1}^{t_0} I_\alpha(t+t_0) I_\beta(t+t_0)\right]_{\rm av}, \quad (17)$$

where t_0 is the number of equilibration sweeps as well the number of measurement sweeps and the subscripts α and β denote independent replicas. When $t_0 \ll t_{\rm eq}$ the spin configurations of the two replicas are completely uncorrelated but as t_0 increases they become more correlated, since they both feel the same random interactions. Thus we expect $I^2(t_0)$ to monotonically increase from zero to the equilibrium value as $t_0 \to t_{\rm eq}$; see Fig. (1). Each of the equilibration times in Table I is chosen to be the least number of sweeps necessary to equilibrate at the lowest temperature, $T_{\rm min}$. For each size except $L = 12$ we ran some samples for $t_0 \gg t_{\rm eq}$ in order to confidently determine $t_{\rm eq}$; the remaining samples were run with $t_0 = t_{\rm eq}$. We should note that for $L = 12$ it was impractical to run jobs for t_0 much greater than the value at which the $I^2(t_0)$ seemed to stop increasing, so we took this value to be $t_{\rm eq}$.

L	$T_{\min}$	$T_{\max}$	n_T	$t_{\rm eq}$	Samples
4	0.3	0.97	11	1280	8000
6	0.3	0.92	18	10240	10937
8	0.3	0.92	27	20480	5388
12	0.3	0.90	47	163840	781

TABLE I. Parameters of the exchange Monte Carlo simulations for each value of L, where $T_{\min}$ and $T_{\max}$ are the minimum and maximum of n_T temperatures, and $t_{\rm eq}$ is the number of sweeps for equilibration, which, in our simulations, is also equal to the number of sweeps for measurements. The number of samples studied for each size is also shown.

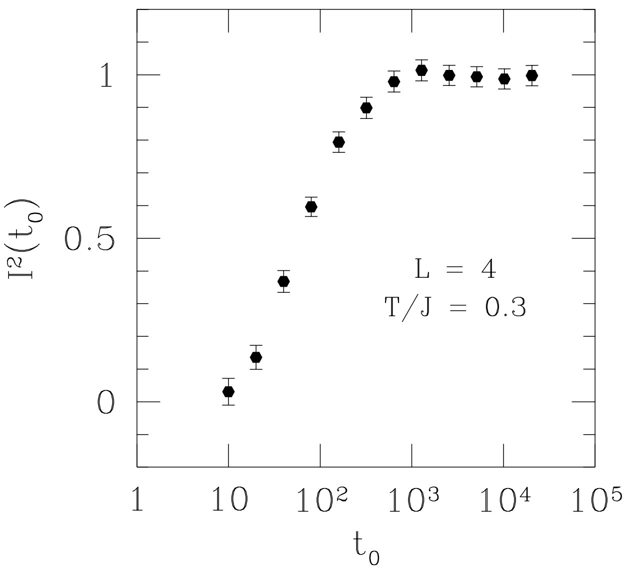

FIG. 1. A semi-log plot of $I^2(t_0)$ against t_0, from Eq. (17) for $L = 4$ at $T/J = 0.3$ using 2000 samples to test for equilibration. One sees that the results do not change for t_0 greater than about 1000. Hence, in the production runs, the equilibration time was taken to be $t_0 = 1280$.

V. RESULTS FOR STATICS

We present our results for $I_{\rm rms}$ from Eq. (10) in Fig. (2). The data cross at $T_c/J = 0.47$ and splay out clearly on *both* sides of the transition. This is the first time that the splaying out of the data for $I_{\rm rms}$ in the ordered state can be distinguished well beyond the size of the error bars, thus presenting incontrovertible evidence for a spin-glass ordered phase in three dimensions. The data for the lowest temperature are given numerically in Table (II). We obtain an uncertainty of 0.03 for T_c/J by setting the error equal to the region over which the data for all sizes overlap within their error bars. Our final estimate for the critical temperature is

$$T_c/J = 0.47 \pm 0.03, \tag{18}$$

which agrees with the previous value of 0.45 ± 0.05 from Reger et al.[13] The key difference is that in Reger et al. the values for $I_{\rm rms}$ did not splay out significantly below T_c and so they did not find compelling evidence for spin glass order.

$L = 4$	$L = 6$	$L = 8$	$L = 12$
1.026 ± 0.009	1.090 ± 0.008	1.165 ± 0.012	1.238 ± 0.037

TABLE II. $I_{\rm rms}$ from Eq. (10) for $T/J = 0.3$ and different L. The values are clearly distinct beyond the size of the error-bars.

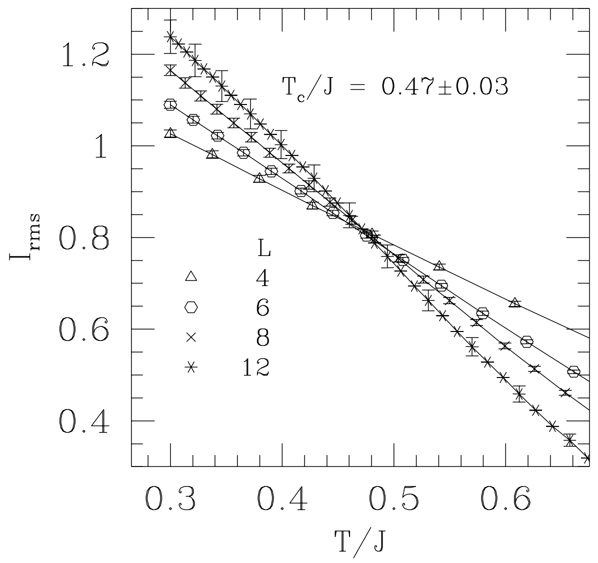

FIG. 2. A plot of $I_{\rm rms}$ from Eq. (10) for $L = 4, 6, 8,$ and 12. For the point of intersection we estimate that the critical temperature is 0.47 ± 0.03. Some of the $L = 12$ errorbars have been removed for clarity.

We obtain an estimate for the correlation length exponent ν by finite-size scaling the data from Fig. (2); see also Eq. (12). Fig. (3) shows that the data scales well with $\nu = 1.39$. This value is obtained by by scaling the data using different choices for ν, fitting a polynomial, calculating the chi-squared statistic for each choice, and minimizing the chi-squared. The error in ν is determined by varying the value of ν until the $L = 8$ data no longer scales with the smaller sizes within the sum of their errorbars. $L = 8$ was chosen rather than $L = 12$ because the errorbars of the former are much smaller. This leads to our estimate

$$\nu = 1.39 \pm 0.20. \tag{19}$$

Variations in T_c from Eq. (18) did not significantly increase the error bar beyond what that shown in Eq. (19). Our result agrees with, and is a bit more precise than, those previously presented by Wengel and Young,[30] $\nu = 1.3 \pm 0.3$, and by Reger et al,[13] 1.3 ± 0.4.

The correlation length exponent has also been deduced from experimental measurements. It is now known,[3,4] however, that the response of the resistivity and critical temperature to tilting of the applied field distinguishes a vortex-glass transition, in which point disorder dominates, from a Bose-glass transition, in which correlated disorder, such as twin boundaries or columnar pins, dominates. To our knowledge, there are only two experiments[6,37] which observe a vortex glass transition *and* demonstrate the proper response to magnetic field tilting.

In the first, Petrean et al.[6] find that for untwinned proton-irradiated $YBa_2Cu_3O_{7-\delta}$ near criticality the resistivity decreases from its maximum as the field is tilted away from the c axis,[6], which signals a vortex glass transition, whereas in twinned $YBa_2Cu_3O_{7-\delta}$, they find a Bose-glass transition with the resistivity increasing from its minimum.[4] Petrean et al.,[6] do not obtain ν but do obtain the resistivity exponent s by fitting resistivity vs. temperature curves. We will compare our results to these in §VI.

In the second work, Klein et al.,[37] study the vortex glass transition in $(K, Ba)BiO_3$. The dependence of their data on field tilting is that expected for the vortex glass, and they obtain $\nu = 1.0 \pm 0.2$ which agrees with ours within the errorbars.

Kawamura[33] has recently modified the gauge glass to include a net magnetic field. He obtains $\nu = 2.2 \pm 0.4$ which is greater than our estimate and the experimental result of Klein et al[37].

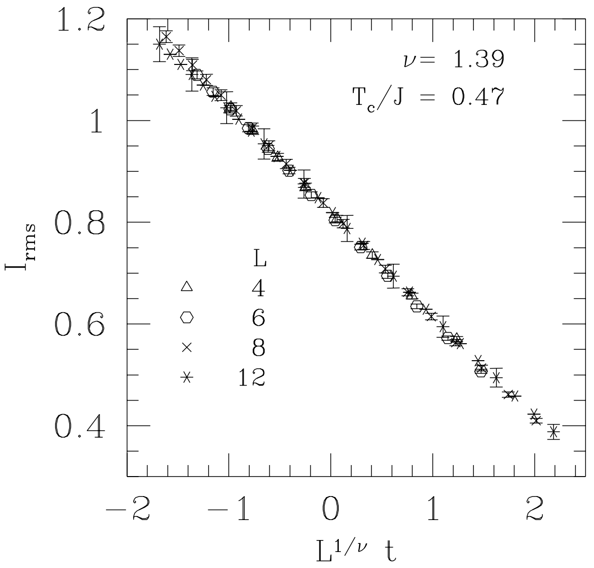

FIG. 3. A plot of $I_{\rm rms}$ from Eq. (10) versus $L^{1/\nu}t$, where $t = (T - T_c)/T_c$ and $\nu = 1.39$. Some of the $L = 12$ errorbars have been removed for clarity.

In addition to ν, we calculate the independent exponent η which describes the decay of the correlation function at criticality. To our knowledge, this is the first calculation of η for the gauge glass in three dimensions. Finite size scaling predicts that, at the critical point, the spin glass susceptibility

$$\chi_{\rm SG} = N \left[\langle |q|^2 \rangle \right]_{\rm av} \tag{20}$$

scales as

$$\chi_{\rm SG} \sim L^{2-\eta}, \tag{21}$$

assuming the hyperscaling relation $\gamma/\nu = 2 - \eta$. We use standard Monte Carlo to calculate the susceptibility at $T/J = 0.44, 0.47,$ and 0.50; the details of these simulations are discussed in §VI. We present our results in Fig. (4). From linear least-squares fits to the data on a log-log scale we obtain

$$\eta = -0.47 \pm 0.07. \tag{22}$$

The error in our estimate comes mainly from the uncertainty in T_c. Kawamura's[33] anisotropic gauge glass yields a similar estimate, $\eta = -0.5 \pm 0.2$.

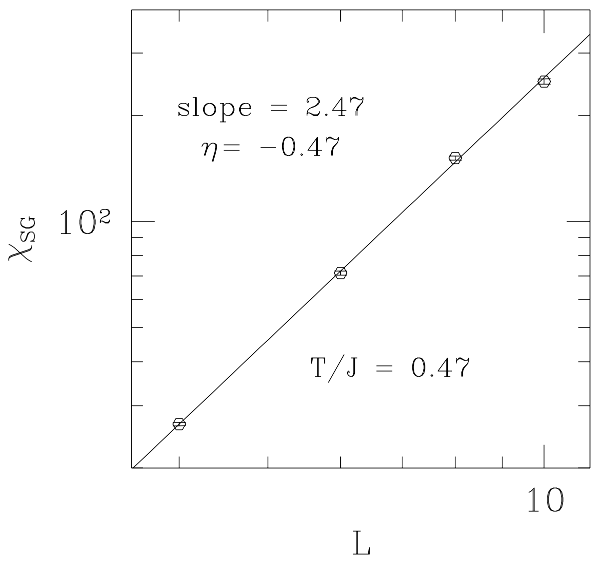

FIG. 4. A log-log plot of $\chi_{\rm SG}$ vs. L for $L = 4, 6, 8,$ and 10 at $T/J = 0.47$ according to Eq. (21) with $\eta = -0.47$.

There has been some controversy regarding the value of the zero temperature stiffness exponent θ. This is typically computed from the root-mean-square "defect energy" on changing the boundary conditions from periodic to anti-periodic, but is also given by the response to an infinitesimal twist in the boundary conditions, as shown in Eq. (11). A positive value for θ indicates a finite temperature transition to a spin glass state, whereas $\theta < 0$ implies $T_c = 0$. Some studies[13,14,17,18] obtain $0 \le \theta \le 0.077$; we refer to these as the low group. Others,[15,16,19] however, calculate $0.26 \leq \theta \leq 0.31$; we name these the high group.

Our results for the rms current are not at sufficiently low T and large L to get a firm estimate for θ, but we obtain a plausible bound as follows. For each temperature below T_c we do a linear least squares fit of $\log I_{\rm rms}$ against $\log L$ to get an *effective* temperature dependent value, $\theta_{\rm eff}(T)$. The results are shown in Fig. (5). We see that $\theta_{\rm eff}$ increases monotonically as T decreases, so we expect that the asymptotic value is greater than the value of $\theta_{\rm eff}$ at the lowest temperature, i.e. we expect

$$\theta \geq 0.18, \tag{23}$$

which is consistent with the results of the "high group".

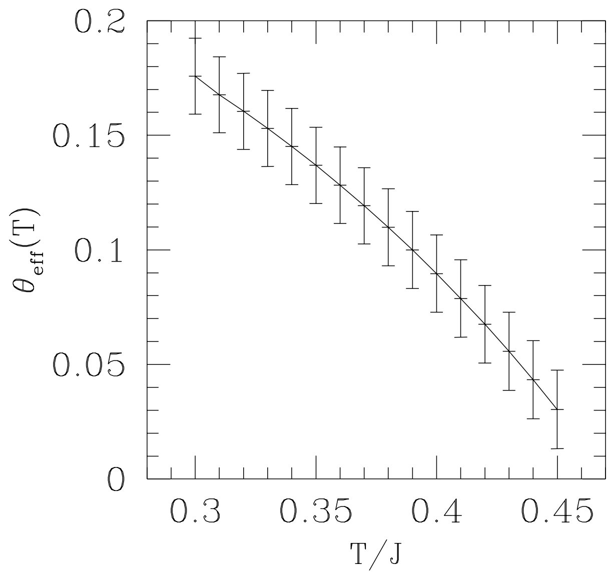

FIG. 5. A plot of $\theta_{\rm eff}(T)$ vs. T. At each T, $\theta_{\rm eff}(T)$ is obtained from a linear least squares fit of $\log I_{\rm rms}$ against $\log L$.

VI. DYNAMICS

When the gauge glass is sufficiently close to criticality the correlation length is of order L and the system experiences critical slowing down. The equilibration time $t_{\rm eq}$ then scales as

$$t_{\rm eq} \sim L^z, \tag{24}$$

where z is the dynamic exponent. At temperatures just above the vortex glass transition, the resistivity ρ is predicted[8] to scale as

$$\rho \sim (T - T_c)^s, \tag{25}$$

where s is related to other exponents by

$$s = \nu(z-1) \tag{26}$$

in three dimensions. We are thus motivated to calculate z to compare our results to experimental measurements of s. It must be emphasized, however, that there are more dynamical universality classes than static universality classes. In our simulations which determine z we use dissipative (Monte Carlo) dynamics with standard Metropolis-type updating probabilities, and *without* the temperature swapping (exchange Monte Carlo) that we used in our simulations of static quantities. Tests for equilibration were carried out as in Bhatt and Young[10].

T/J	$L=4$	$L=6$	$L=8$	$L=10$
0.44	500	400	358	372
0.47	500	400	328	246
0.50	500	400	350	321

TABLE III. Number of samples in the standard Monte Carlo simulations used for calculating the dynamic exponent. Three temperatures were used to account for the uncertainty in T_c.

The dynamic exponent can be obtained by a finite size scaling of time-dependent measurements of the spin glass susceptibility. We use the "two-replica" susceptibility $\chi_{\rm SG}(t_0)$ defined by

$$\chi_{\rm SG}(t_0) = \left[\frac{1}{N t_0} \sum_{t=1}^{t_0} \left| \sum_{j=1}^{N} e^{i[\phi_j^\alpha(t_0+t) - \phi_j^\beta(t_0+t)]} \right|^2 \right]_{\rm av} \tag{27}$$

where α and β denote independent replicas.

We obtain our estimate of z by a finite-size scaling analysis of the data for $\chi_{\rm SG}(t_0)$ for sizes $L = 6, 8,$ and 10; the scaling is better without the $L = 4$ data, so it has been omitted. One expects from Eq. (24) that at criticality

$$\frac{\chi_{\rm SG}(t_0)}{\chi_{\rm SG}(t_{\rm eq})} = f(L^{-z} t_0) \tag{28}$$

where f is a scaling function. Our results, shown in Fig. (6), yield

$$z = 4.2 \pm 0.6, \tag{29}$$

as calculated from the $T/J = 0.47$ data. The estimate of the uncertainty is obtained by varying z until the $L = 10$ data no longer scales with the smaller sizes within the errorbars. Similar scaling plots from the $T/J = 0.44$ and $T/J = 0.50$ yield z values within Eq. (29), thus the uncertainty in T_c is not the dominant contribution to the error in z. Our value agrees with the previous estimate $z = 4.7 \pm 0.7$ by Reger et al.[13] and $z = 3.3 \pm 0.5$ from Kawamura's[33] anisotropic gauge glass.

Using Eqs. (29) and (19), the resistivity exponent is obtained from Eq. (26),

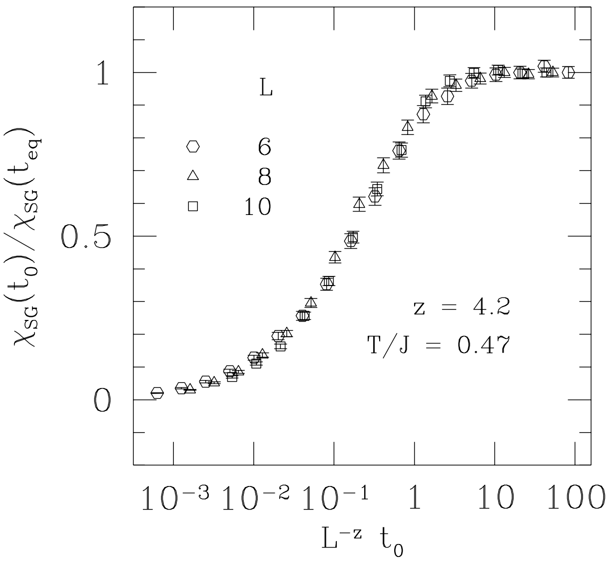

FIG. 6. A finite-size scaling plot of $\chi_{SG}(t_0)$ using Eq. (28) for $L = 6, 8,$ and 10 at $T/J = 0.47$ with $z = 4.2$.

$$s = 4.5 \pm 1.1 \tag{30}$$

This value agrees with the experiments of Klein et al.[37], $s = 3.9 \pm 0.3$, on $(K, Ba)BiO_3$, as well as Petrean et al.,[6] $s = 5.3 \pm 0.7$, who study untwinned proton-irradiated $YBa_2Cu_3O_{7-\delta}$.

VII. CONCLUSIONS

In this work we have presented exchange Monte Carlo results for the gauge glass model in three dimensions. We have shown for the first time incontrovertible evidence of a vortex-glass ordered phase below T_c. To our knowledge, this is the first time that such clear ordering has been shown for *any* spin-glass model in three dimensions. The correlation length exponent ν has been calculated to higher precision than before. We have also obtained the first estimate of the correlation function exponent η from Monte Carlo simulations. Finally, our values of ν and z are combined to estimate the resistivity exponent s. Our results are summarized in Table (IV).

It is interesting that our values of η and ν agree with those of two other glassy systems, the three-dimensional $\pm J$ Ising spin glass[11] and chiral ordering of the three-dimensional $\pm JXY$ spin glass.[38] Kawashima and Young[11] obtain $\nu = 1.7 \pm 0.3$ and $\eta = -0.35 \pm 0.05$ for the Ising spin glass, while Kawamura[38] obtains $\nu = 1.5 \pm 0.3$ and $\eta = -0.4 \pm 0.2$ for the chiral glass ordering. However, it is expected that the Ising and gauge glass models are in different universality classes because the order parameter has a different number of components in each case: one for the Ising spin glass, and two (the real and imaginary parts of Eq. (4)) for the gauge glass. Indeed, a first order $\epsilon = 6 - d$ expansion shows that the gauge glass exponents are not the same as any n-component vector spin glass.[39] The order parameters for the Ising spin glass and the chiral transition in the XY spin glass do have the same dimensionality, but the transitions are still likely to be in different universality classes because of the long range interaction between chiralities in the XY spin glass. The error bars in the exponents are not extremely small so the apparent agreement between the results may be simply a numerical coincidence.

T_c/J	ν	η	z
0.47 ± 0.03	1.39 ± 0.20	-0.47 ± 0.07	4.2 ± 0.6

TABLE IV. Critical temperature and exponents of the gauge glass in three dimensions.

To confidently compare these results with experiment one has to show that the critical point in the gauge glass model is in the same universality class as an anisotropic model with a net field. Clearly, if such anisotropy causes the scaling behavior to be anisotropic, in the sense that the *exponents* for the divergence of the correlation length are different for separations along and perpendicular to the field, then it is relevant. If, however, the anisotropy induced by a net field does not lead to anisotropic scaling, but just causes the *amplitudes* of the correlation lengths in the different directions to be different, then it is not clear why it should be relevant, just as making an Ising ferromagnet anisotropic by having the bonds in one direction different from those in the other directions does not change the universality class.

Kawamura[33] has studied a vortex glass model with a net field and finds different critical behavior from that of the gauge glass, even though the scaling is *not* anisotropic in the sense defined above. However, he imposes free boundary conditions which may lead to very large corrections to finite-size scaling (since a large fraction of the sites are on the boundary). Hence it would be very useful to study the vortex glass transition in a model with a net field *and* periodic boundary conditions.

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