Numerical studies of the two- and three-dimensional gauge glass at low temperature

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(Dated: November 20, 2002)

We present results from Monte Carlo simulations of the two- and three-dimensional gauge glass at low temperature using the parallel tempering Monte Carlo method. Our results in two dimensions strongly support the transition being at $T_c = 0$. A finite-size scaling analysis, which works well only for the larger sizes and lower temperatures, gives the stiffness exponent $\theta = -0.39 \pm 0.03$. In three dimensions we find $\theta = 0.27 \pm 0.01$, compatible with recent results from domain wall renormalization group studies.

PACS numbers: 75.50.Lk, 75.40.Mg, 05.50.+q

I. INTRODUCTION

The gauge glass is a model which is often used to describe the vortex glass transition in high-temperature superconductors. Consequently there has been a substantial amount of theoretical work attempting to understand its properties. Nonetheless, there are still areas of disagreement. For example, in two dimensions there is an ongoing controversy as to whether a spin-glass transition occurs at finite temperature or not. In addition, in three dimensions, although it is known that the system exhibits a finite-temperature transition, see, e.g., Ref. 1, there is no consensus on the value of the stiffness exponent.

Recent results by means of resistively-shunted junction (RSJ) dynamics by Kim[2] claim evidence for a finite-temperature transition in two dimensions with a transition temperature $T_c = 0.22$. Choi and Park[3] find the same value for T_c by studying the scaling of the spin-glass susceptibility via Monte Carlo simulations. These claims for a finite T_c are in contrast to results by Granato[4] and Hyman *et al.*[5] who also use RSJ dynamics and find evidence of a zero-T transition. In addition, Monte Carlo simulations by Fisher *et al.*[6] and Reger and Young[7] show evidence of a zero-temperature transition, although the simulations were not taken down to extremely low temperatures.

In this work we perform Monte Carlo simulations of the two-dimensional gauge glass, using parallel tempering[8,9] to go to significantly lower temperatures than was possible in earlier work. In particular we are now able to cover the temperature range where the claimed spin-glass transition[2,3] takes place. We find strong evidence that $T_c = 0$.

In addition, we study the gauge glass in three dimensions at very low, but finite, temperatures to provide a good estimate of the stiffness exponent θ. The motivation for this is that earlier estimates, which were obtained from ground state methods, are inconsistent. One group[10–13] finds values consistent with $\theta \approx 0$, whereas another[14–17] finds θ in the range 0.26 – 0.31. Ground state methods involve computing the ground state with two different boundary conditions, and one of the main reasons for the large range of estimates for θ is that different groups made different assumptions about the optimal form of boundary condition changes, see, e.g., Ref. 14. This difficulty is avoided in the finite-T Monte Carlo approach adopted here. We find that $\theta = 0.27 \pm 0.01$, which agrees with the results of the second group of ground state calculations mentioned above.

In Sec. II we introduce the model and observables as well as details regarding the Monte Carlo technique used. Some results from finite-size scaling are discussed in Sec. III. Our results in two and three dimensions are presented in Secs. IV and V, respectively, and the conclusions are summarized in Sec. VI.

II. MODEL, OBSERVABLES AND EQUILIBRATION

The Hamiltonian of the gauge glass is given by

$$\mathcal{H} = -J \sum_{\langle i,j \rangle} \cos(\phi_i - \phi_j - A_{ij}), \tag{1}$$

where the sum ranges over nearest neighbors on a square lattice in D dimensions of size $N = L^D$ and ϕ_i represent the angles of the XY spins. Periodic boundary conditions are applied. The A_{ij} are quenched random variables uniformly distributed between $[0, 2\pi]$.[18] Because A_{ij} represent the line integral of the vector potential between sites i and j, we have the constraint that $A_{ij} = -A_{ji}$. In this work we set $J = 1$.

Traditionally one uses the Binder ratio[19] to estimate the critical temperature T_c. In this method one plots the ratio of the fourth moment and the second moment squared of the spin-glass order parameter as a function of temperature for different system sizes L. The crossing point of all curves identifies T_c. For the gauge glass the

Binder ratio cannot exceed unity,[1] thus the splaying of the curves is small and T_c is difficult to establish.

In order to avoid this problem we use a method introduced by Reger and Young[10] in which one calculates the current I defined as the derivative of the free energy F with respect to an infinitesimal twist to the boundaries, i.e.,

$$I(L) = \frac{1}{L}\sum_i \sin(\phi_i - \phi_{i+\hat{x}} - A_{i\,i+\hat{x}}) \, . \qquad (2)$$

In this case the twist is applied along the $\hat{x}$ direction. As the gauge fields A_{ij} are uniformly distributed we expect $[\langle I(L)\rangle]_{\rm av} = 0$, where $[\cdots]_{\rm av}$ represents an average over disorder and $\langle\cdots\rangle$ represents a thermal average. We calculate the root-mean-square current

$$I_{\rm rms} = \sqrt{[\langle I_\alpha(L)\rangle\langle I_\beta(L)\rangle]_{\rm av}} \, . \qquad (3)$$

Here α, β denote two replicas of the system with the *same* disorder introduced to avoid any bias in the calculation of the average of the square of the currents. Unlike the Binder ratio, the currents are not restricted to converge to unity for $T \to 0$, thus any potential splaying of the data is easily visible. In fact, for $T < T_c$ the currents scale as $I_{\rm rms} \sim L^\theta$ where θ is the (positive) stiffness exponent, see Refs. 20–22 for a general discussion of the stiffness exponent in the context of the "droplet model" of spin glasses, and also Ref. 23 which discusses the stiffness exponent specifically in the context of the vortex glass.

Note that the stiffness exponent is also often defined in terms of the free energy change for a *finite* twist, by π for example. We are assuming here that any reasonable measure of the stiffness will give the same exponent θ, whether it be the free energy change due to a finite twist or, as here, the derivative with respect to an infinitesimal twist. This is consistent with the usual assumptions of universality for phenomena on long length scales, that the same exponent will be obtained for the relation between, say, energy and length scale, for any reasonable definition of energy. Presumably a plot of the free energy against twist angle Θ will have quite complicated structure, with local regions varying quadratically as a function Θ and proportional to L^{d-2} (where only spin-wave excitations are generated), but with "kinks" where which the system changes from one parabola to another due to a vortex changing position. Hence, in effect, we are assuming that the typical magnitude of the slope of this curve is of the same order in L as the range of its variation with Θ. This requires that the typical spacing between kinks $\Delta\Theta$ satisfies $L^\theta \sim (\Delta\Theta)^2 L^{d-2}$, i.e. $\Delta\Theta \sim L^{-(d-2-\theta)/2}$.

For a finite-temperature transition, $T_c > 0$, we expect $\theta > 0$, since then the ordered state at $T = 0$ is "stiff" on large scales, and so will presumably resist small thermal fluctuations. On the other hand for a zero-temperature transition, we expect $\theta < 0$, because the system will then easily break up under the influence of thermal fluctuations.

In order to compare with the results of Choi and Park[3] we have also calculated the spin-glass susceptibility $\chi_{\rm SG}$ defined by

$$\chi_{\rm SG} = N[\langle q^2\rangle]_{\rm av} \, , \qquad (4)$$

where q is the spin-glass order parameter defined by

$$q = \frac{1}{N}\sum_j^N \exp[i(\phi_j^\alpha - \phi_j^\beta)] \qquad (5)$$

and α, β are two replicas of the system with the same disorder. This susceptibility diverges at the transition. However, as we shall see in Sec. III, its finite-size scaling behavior is more complicated than that of $I_{\rm rms}$.

For the simulations we use the parallel tempering Monte Carlo method[8,9] as it allows us to study systems at lower temperatures than with conventional methods. Because the equilibration test for short-range spin glasses introduced by Katzgraber *et al.*[24] does not work for the gauge glass since the disorder is not Gaussian, we test equilibration by the traditional technique of requiring that different observables are independent of the number of Monte Carlo steps $N_{\rm sweep}$. Figure 1 shows data in 2D for the energy E, spin-glass susceptibility $\chi_{\rm SG}$, as well as the average current squared $I_{\rm rms}^2$ as a function of Monte Carlo steps. One can clearly see that the different observables saturate at the *same* equilibration time. We show data for an intermediate size as it allows us to better illustrate the procedure by simulating with much longer runs than are necessary to equilibrate. For large sizes we equilibrate doubling the number of Monte Carlo sweeps between each measurement until the last three agree within error bars.

We also require that the acceptance ratios for the global moves which interchange the different temperatures in the parallel tempering scheme be greater than 0.3 on average and roughly constant as a function of temperature. The equilibration tests for the 3D data have been described elsewhere.[18]

Tables I and II show the number of samples $N_{\rm samp}$ and the number of Monte Carlo sweeps $N_{\rm sweep}$ performed by each replica for each lattice size for $D = 2$ and $D = 3$, respectively. In two dimensions the highest temperature is 1.058, whereas the lowest temperature is 0.13 (for $L = 24$ the lowest temperature is 0.20). The number of temperatures N_T is chosen to give satisfactory acceptance ratios for the Monte Carlo moves between the temperatures. In three dimensions the lowest temperature studied is 0.05 [to be compared with $T_c \approx 0.45$, (Ref. 1)] whereas the highest temperature is 0.947.

Because the gauge glass has a vector order parameter symmetry, to speed up the simulations we discretize the angles of the spins[15,18] to $N_\phi = 512$. To ensure a reasonable acceptance ratio for single-spin Monte Carlo moves, we pick the proposed new angle for a spin within a temperature-dependent acceptance window about the current angle. By tuning a numerical prefactor we ensure the acceptance ratios for these local moves are not

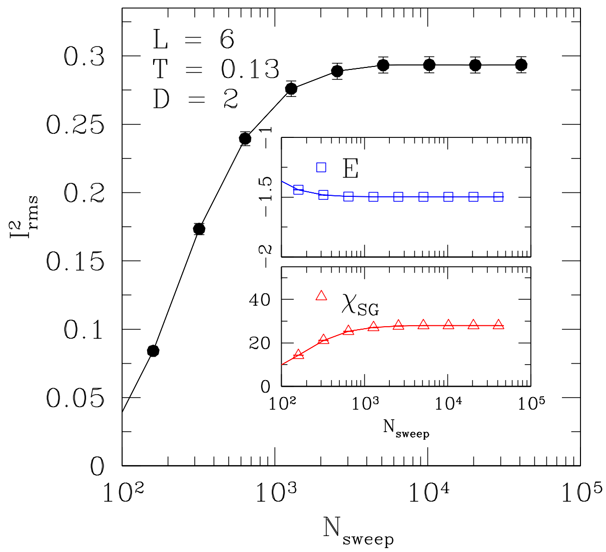

FIG. 1: Average current squared $I_{\rm rms}^2$, energy E and spin-glass susceptibility $\chi_{\rm SG}$ as a function of Monte Carlo sweeps $N_{\rm sweep}$, that each of the replicas perform, averaged over the last half of the sweeps. Note that the different observables equilibrate roughly at the same time and appear to be independent of the number of sweeps. The data shown is for $D = 2$, $L = 6$ and $T = 0.13$, the lowest temperature studied in two dimensions.

TABLE I: Parameters of the simulation in two dimensions. $N_{\rm samp}$ is the number of samples, i.e., sets of disorder realizations, $N_{\rm sweep}$ is the total number of sweeps simulated for each of the $2N_T$ replicas for a single sample, and N_T is the number of temperatures used in the parallel tempering method. For $L \le 16$ the lowest temperature is 0.13, while for $L = 24$ it is 0.20.

L	$N_{\rm samp}$	$N_{\rm sweep}$	N_T
4	10400	8.0×10^4	30
6	10150	8.0×10^4	30
8	8495	2.0×10^5	30
12	6890	8.0×10^5	30
16	2500	2.0×10^6	30
24	2166	2.0×10^6	24

smaller than 0.2 for each system size at the lowest temperature simulated.

III. FINITE-SIZE SCALING

We review briefly some salient aspects of finite-size scaling theory, as applied to the present situation, paying particular attention to features which are special to a $T = 0$ transition. Consider first the singular part of the *bulk* free energy per spin divided by T, a quantity with dimension (length)$^{-D}$, where D represents the space dimension. The basic assumption is that the only relevant lengthscale is the correlation length ξ, and so $\beta f_s \sim \xi^{-D}$.

TABLE II: Parameters of the simulation in three dimensions. The lowest temperature studied is $T = 0.05$.

L	$N_{\rm samp}$	$N_{\rm sweep}$	N_T
3	10000	6.0×10^3	53
4	10000	2.0×10^4	53
5	10000	6.0×10^4	53
6	5000	2.0×10^5	53
8	2000	1.2×10^6	53

Consider next a finite-size system where the bulk behavior is assumed to be modified by a function of L/ξ, where L is the lattice size, i.e.,

$$\beta f_s = \xi^{-D} \tilde{f} \left(\frac{L}{\xi} \right) . \tag{6}$$

Thus, we can write the scaling of the total singular free energy $F_s (= L^D f_s)$ as

$$\beta F_s = \tilde{F} \left(\frac{L}{\xi} \right) = \hat{F}(L^{1/\nu}(T - T_c)) . \tag{7}$$

The last expression follows because $\xi \sim (T - T_c)^{-\nu}$ and we have taken the argument of the function $\tilde{F}$ to the power $1/\nu$.

As $I_{\rm rms}$ is the derivative of F_s with respect to the twist angle and since the twist angle is dimensionless, the scaling of $I_{\rm rms}$ is the same as that of F_s, i.e.,

$$\beta I_{\rm rms} = \hat{I}[L^{1/\nu}(T - T_c)] . \tag{8}$$

Note that we have been careful to maintain the factor of β in our analysis. However, if $T_c > 0$, then, since the critical region is close to T_c, we can replace β by $1/T_c$ and incorporate this *constant* factor into the scaling function, i.e.,

$$I_{\rm rms} = \tilde{I}[L^{1/\nu}(T - T_c)] \qquad (T_c > 0). \tag{9}$$

By contrast, if $T_c = 0$ we have to keep the variation in β and so

$$\begin{aligned} I_{\rm rms} &= T\hat{I}(L^{1/\nu}T) \\ &= L^{-1/\nu}\tilde{I}(L^{1/\nu}T) \qquad (T_c = 0) , \end{aligned} \tag{10}$$

where the scaling functions $\hat{I}$ and $\tilde{I}$ are simply related to each other. Equation (10) indicates that the $T = 0$ stiffness exponent θ is negative and equal to $-1/\nu$. Equation (9) shows that if $T_c > 0$ the curves for $I_{\rm rms}$ for different sizes intersect at the critical point, whereas Eq. (10) shows that if $T_c = 0$, then the data decrease with increasing size at $T = 0$.

According to standard finite-size scaling the spin-glass susceptibility, defined in Eq. (4), behaves as

$$\chi_{\rm SG} = L^{2-\eta}\tilde{\chi}_{\rm SG}[L^{1/\nu}(T - T_c)] , \tag{11}$$

which means that at criticality it diverges with a power law, i.e.,

$$\chi_{\rm SG} \sim L^{2-\eta} \qquad (T = T_c) . \tag{12}$$

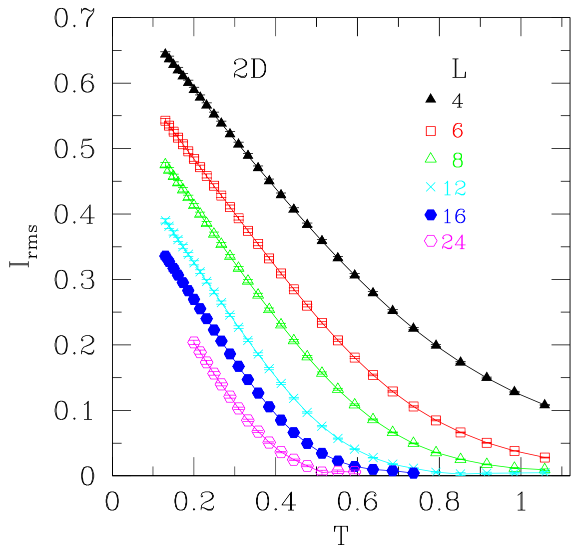

FIG. 2: Root-mean-square current I_{rms} in two dimensions as a function of temperature for different system sizes. At all temperatures the data decrease with increasing L indicating, from Eq. (9), that if T_c is finite it must be less than 0.13.

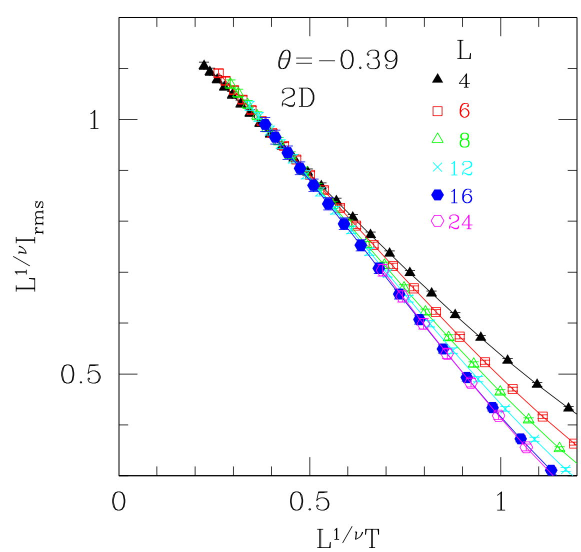

FIG. 3: Scaling plot of the root-mean-square current I_{rms} in two dimensions according to the form expected if $T_c = 0$, Eq. (10). We see acceptable scaling of the data at low temperatures. Deviations at higher T are presumably due to corrections to scaling. This plot is for $\theta \equiv -1/\nu = -0.39$.

Equations (11) and (12) are valid whether the transition is at $T_c = 0$ or $T_c > 0$. The power law prefactor in Eq. (11) with an unknown exponent complicates the analysis of χ_{SG} compared with that for I_{rms} which does not have such a prefactor if $T_c > 0$, see Eq. (9). Even if $T_c = 0$, where a prefactor does arise [see Eq. (10)], the exponent in the prefactor is the *same* as that in the argument of the scaling function, so we do not have an *extra* exponent to determine.

IV. RESULTS FOR $D = 2$

In Fig. 2 we present results for I_{rms} in two dimensions. At all temperatures the data decrease with increasing L indicating, from Eq. (9), that T_c must be less than the range of temperatures studied. In fact, we shall find it impossible to scale the data for I_{rms} for *any* finite T_c, in contrast to the results of Kim[2] and Choi and Park,[3] who find a finite-temperature transition at $T_c = 0.22$.

In Fig. 3 we show a finite-size scaling plot of the data in Fig. 2 according to Eq. (10). Although the data do not scale well over the whole range, the data at low-T *do* scale quite well, since the plot shows that the data collapse if $L^{1/\nu}T$ is small for all values of L, and over the whole range of $L^{1/\nu}T$ for the largest sizes. The data in Figs. 2 and 3 are therefore consistent with a zero temperature transition, but with significant corrections to scaling at intermediate temperatures. We estimate the stiffness exponent to be

$$\theta = -0.39 \pm 0.03\ . \tag{13}$$

The above error bar is estimated by varying θ slightly until the data do not collapse well. This result is consistent with recent work of Akino and Kosterlitz[14] who find $\theta = -0.36 \pm 0.01$.

Figure 4 shows the current data according to Eq. (9) assuming the parameters found by Choi and Park[3]: $T_c = 0.22$, $1/\nu = 0.88$. The scaling is *very* poor especially near the proposed T_c. By contrast, if the deviations are due to corrections to scaling, they should be smaller near T_c. Hence our data are incompatible with the claim of Kim[2] and Choi and Park[3] that $T_c = 0.22$. In fact, we are unable to get a reasonable fit to the data for I_{rms} according to Eq. (9) for *any* finite T_c.

Next we present our data for the spin-glass susceptibility χ_{SG}. Figure 5 is a log-log plot of χ_{SG} vs L at several low temperatures. According to Eq. (12) the data should lie on a straight line at T_c. However, the data in the vicinity of $T = 0.22$, the transition temperature claimed by Kim[2] and Choi and Park,[3] show a strong downward curvature, indicating that this is actually *above* T_c. Only around the lowest temperature where we have data, $T = 0.13$, is the curvature small, although it still greatly exceeds the error bars. This indicates that $T_c < 0.13$, which is compatible with our data for I_{rms}.

Figure 6 shows a scaling plot according to Eq. (11) for $T_c = 0$, and $1/\nu = 0.39$, the same parameters found in the scaling of I_{rms}, together with $\eta = 0$ which is expected at a zero-temperature transition in two dimensions. The data at low temperatures and for the largest sizes scale well, but the data away from this range show deviations. Allowing $1/\nu$ to vary we find $1/\nu = 0.50 \pm 0.03$. The inset shows data for the optimal value, where only the

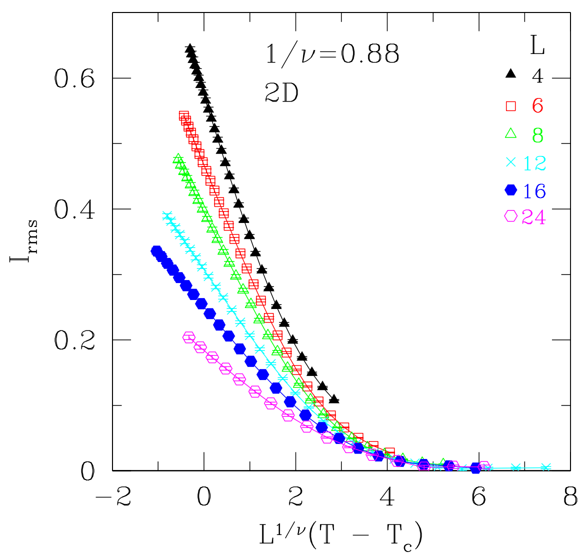

FIG. 4: Scaling of the current according to Eq. (9) with the parameters of Choi and Park (Ref. 3): $T_c = 0.22$, and $1/\nu = 0.88$. One can clearly see that the scaling is very poor, *especially* in the vicinity of the proposed T_c.

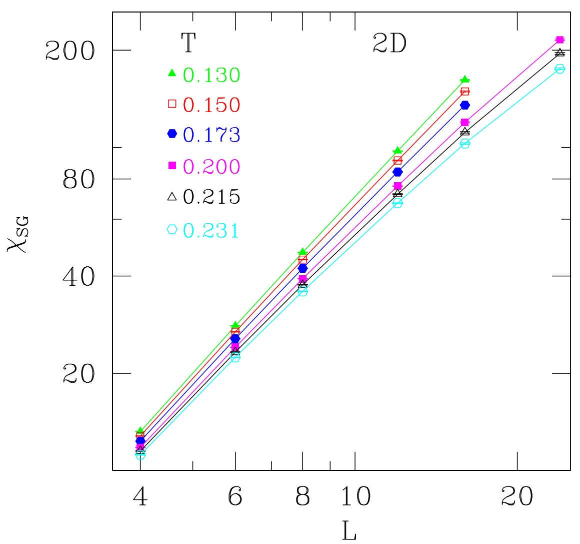

FIG. 5: Log-log plot of the spin-glass susceptibility $\chi_{\rm SG}$ as a function of system size L for several different low temperatures. We clearly see a downward curvature in the data indicating that $T_c < 0.13$.

$L = 4$ and 6 data are not part of the scaling function for all $L^{1/\nu}T$. The fact that the best values of $1/\nu$ are not precisely the same when obtained from $\chi_{\rm SG}$ and $I_{\rm rms}$ presumably indicates that scaling is only valid for fairly low temperatures and large sizes, and that, despite our working at quite low temperatures, we have only a limited range of data which are fully in the scaling regime.

In Fig. 7 we show the same data as in Fig. 6 scaled

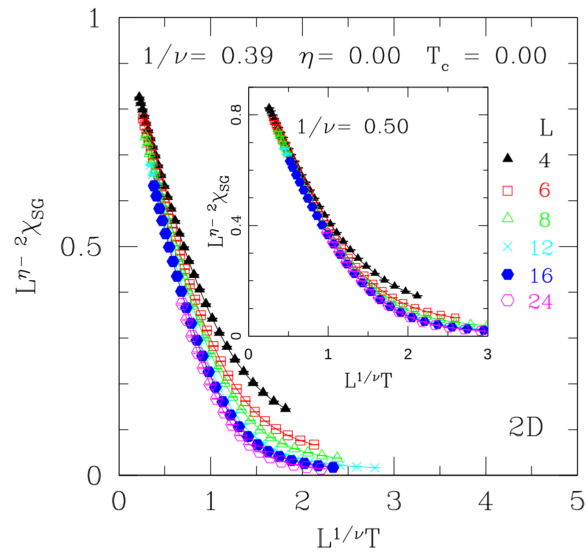

FIG. 6: Scaling of the spin-glass susceptibility $\chi_{\rm SG}$ according to Eq. (11) with $T_c = 0$. The data for large sizes and low temperatures collapse with $\eta = 0$ and $1/\nu = 0.39$. The inset shows a scaling plot for the optimal value $1/\nu = 0.50$ (and $\eta = 0$). For these values of exponents the collapse extends to a larger range of sizes.

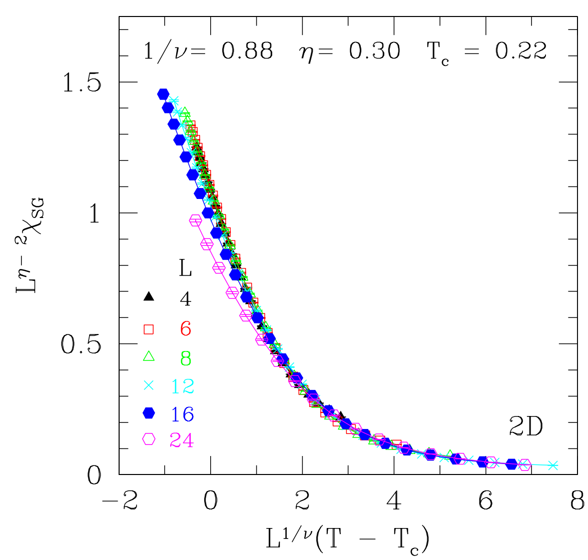

FIG. 7: Scaling of the spin-glass susceptibility $\chi_{\rm SG}$ according to Eq. (11) for $T_c = 0.22$, $1/\nu = 0.88$ and $\eta = 0.30$, as suggested by Choi and Park.[3]

with the parameters used by Choi and Park in Ref. 3. We note that the quality of the data collapse is poor near T_c, whereas if the deviations were due to corrections to scaling, we would expect it to be best in this region. We therefore do not consider this scaling to be acceptable. Furthermore, the scaling of $I_{\rm rms}$ shown in Fig. 4 is clearly much worse, indicating that $I_{\rm rms}$ is much better able to

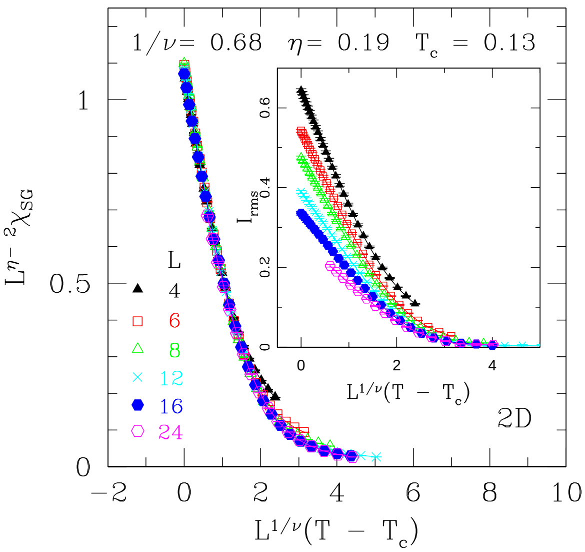

FIG. 8: Scaling of the spin-glass susceptibility $\chi_{\rm SG}$ according to Eq. (11) for $T_c = 0.13$, the lowest temperature in our simulations, $1/\nu = 0.68$ and $\eta = 0.19$. The inset shows the scaling of the $I_{\rm rms}$ according to Eq. (9). One can clearly see that this data do not scale.

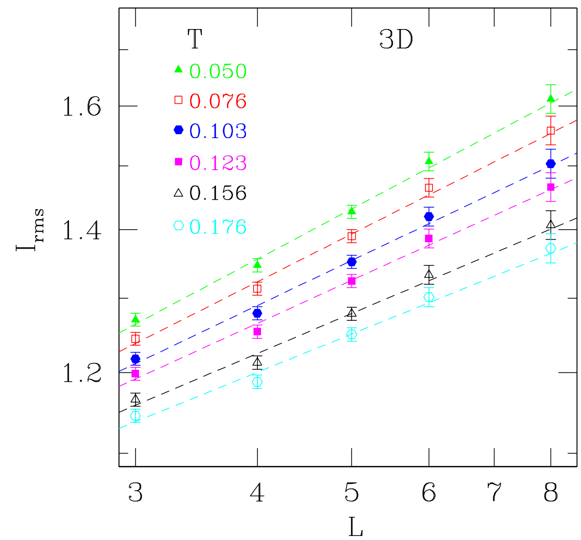

FIG. 9: Log-log plot of $I_{\rm rms}$ vs L for different low temperatures. The data fit well to aL^{θ} with fitting probabilities between 0.57 for the lowest temperature $T = 0.050$ and 0.35 for $T = 0.176$.

distinguish between a finite T_c and $T_c = 0$ than is $\chi_{\rm SG}$ because of its simpler finite-size scaling form.

This is further illustrated in Fig. 8 which shows a scaling plot of $\chi_{\rm SG}$ in which we took T_c to be the lowest temperature in our simulations, i.e., $T_c = 0.13$. The best fit with this T_c has $1/\nu = 0.68$ and $\eta = 0.19$, where η is determined by the requirement that the data scale as well as possible at T_c. By eye, the data scale fairly well, though the deviations are actually significantly greater than the error bars, as we noted in the above discussion of the unscaled $\chi_{\rm SG}$ data at $T = 0.13$. However, the data for I_{rms}, scale very badly with $T_c = 0.13$ as can be seen in the inset of the figure. In fact, as noted above, the data for I_{rms} do not scale according to Eq. (9) for *any* finite T_c.

The data for $\chi_{\rm SG}$ enable us to determine the exponent η as well as $1/\nu$. Assuming $T_c = 0$, as obtained from the $I_{\rm rms}$ data, we find $\eta = 0.0 \pm 0.1$ from a scaling plot for $\chi_{\rm SG}$. This is consistent with $\eta = 0$, as expected at a $T = 0$ transition in two dimensions, at least if the ground state is not highly degenerate.

V. RESULTS FOR $D = 3$

The critical region of the three-dimensional gauge glass has been investigated in detail by Olson and Young.[1] In their work they obtain a lower bound for the stiffness exponent of $\theta \geq 0.18$. Some previous results[10–13] find θ in the range $0 \leq \theta \leq 0.077$ whereas others[14–17] find a much larger value, $0.26 \leq \theta \leq 0.31$.

We can estimate θ from our data for $I_{\rm rms}$ since $I_{\rm rms} \sim L^{\theta}$ when $L^{1/\nu}(T - T_c)$, the argument of the scaling function in Eq. (9), tends to infinity. The data are shown in Fig. 9, in which one observes that the slopes in the log-log plot vary somewhat with temperature. Asymptotically, however, one expects the same L^{θ} behavior for *all* $T < T_c$, so the deviations from a constant slope presumably indicate that that the data are not yet at sufficiently large sizes and low enough temperature to be in the asymptotic region. To obtain an estimate of θ we therefore perform a linear least-squares fit of $\ln I_{\rm rms}$ against $\ln L$ for each temperature in order to obtain an *effective* stiffness exponent $\theta_{\rm eff}(T)$ which depends on the temperature. Figure 10 shows that $\theta_{\rm eff}(T)$ can be fitted well to a linear form at low temperatures. Extrapolating to $T = 0$ we obtain

$$\theta = 0.27 \pm 0.01 \tag{14}$$

which is clearly positive and consistent with the results of Refs. 1,14–17.

VI. CONCLUSIONS

We have shown results from Monte Carlo simulations at low temperatures of the two- and three-dimensional gauge glass. In two dimensions our data for $I_{\rm rms}$ are consistent with a zero-temperature transition with $\theta = -0.39 \pm 0.03$. However, it is necessary to go to quite low temperatures and large sizes to see the expected scaling. Our data are incompatible with the prediction made by Kim[2] and Choi and Park[3] that $T_c = 0.22$. The error bar we quote is purely statistical and systematic corrections

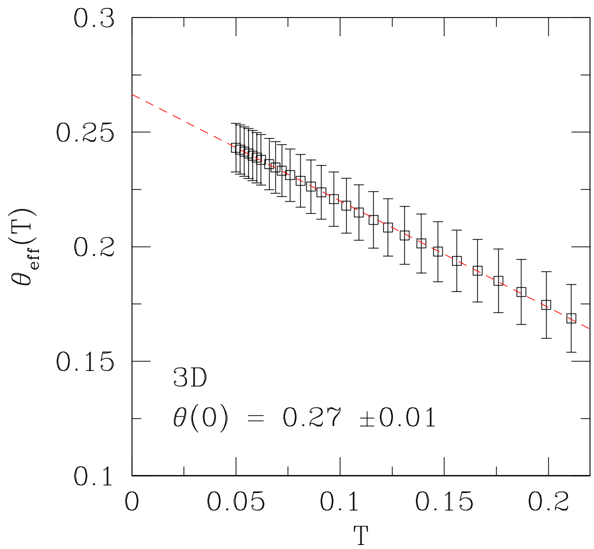

FIG. 10: Effective stiffness exponent $\theta_{\rm eff}(T)$ as a function of temperature in three dimensions for low temperatures. The data extrapolate linearly to $T = 0$.

to scaling can increase this. Indeed, our best estimate for $1/\nu \equiv -\theta$ from the $\chi_{_{\rm SG}}$ data is 0.50 ± 0.03, which differs from the value of $1/\nu$ from $I_{\rm rms}$ by more than the (statistical) error bars. However, our claim that $T_c = 0$ is very robust and is not affected by corrections to finite-size scaling.

In three dimensions we report the first reliable estimate of the stiffness exponent from finite-temperature Monte Carlo simulations. Monte Carlo has the advantage over ground state methods that it computes the stiffness directly and is free from the difficulty of determining the optimal boundary condition changes, which has been quite controversial for the ground state approach. We find $\theta = 0.27 \pm 0.01$, which agrees with the results of Refs. 1,14–17.

Acknowledgments

We would like to thank C. Dasgupta, R. T. Scalettar, P. Sengupta and A. Slepoy for fruitful discussions, and R. T. Scalettar and T. Olson for comments on an earlier version of the manuscript. HGK acknowledges support from the National Science Foundation under Grant No. DMR 9985978. APY acknowledges support from the National Science Foundation under Grant No. DMR 0086287, and the EPSRC under Grant No. GR/R37869/01. He would also like to thank David Sherrington for hospitality during his stay at Oxford. This research was supported in part by NSF cooperative agreement ACI-9619020 through computing resources provided by the National Partnership for Advanced Computational Infrastructure at the San Diego Supercomputer Center and the Texas Advanced Computing Center. We would like to thank the University of New Mexico for access to their Albuquerque High Performance Computing Center. This work utilized the UNM-Alliance Los Lobos Supercluster. We would also like to acknowledge computer time on the UC Davis Undergraduate Computing Cluster and the UCSC Physics Graduate Computing Cluster funded by the Department of Education Graduate Assistance in the Areas of National Need program.

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[1] T. Olson and A. P. Young, *Finite temperature ordering in the three-dimensional gauge glass*, Phys. Rev. B **61**, 12467 (2000), (cond-mat/991229).

[2] B. J. Kim, *Finite-temperature resistive transition in the two-dimensional XY gauge glass model*, Phys. Rev. B **62**, 644 (2000).

[3] M. Y. Choi and S. Y. Park, *Phase transition in the two-dimensional gauge glass*, Phys. Rev. B **60**, 4070 (1999).

[4] E. Granato, *Current-voltage scaling of chiral and gauge-glass models of two-dimensional superconductors*, Phys. Rev. B **58**, 11161 (1998).

[5] R. A. Hyman, M. Wallin, M. P. A. Fisher, S. M. Girvin, and A. P. Young, *Current-voltage characteristics of two-dimensional vortex-glass models*, Phys. Rev. B **51**, 15304 (1995).

[6] M. P. A. Fisher, T. A. Tokuyasu, and A. P. Young, *Vortex variable-range-hopping resistivity in superconducting films*, Phys. Rev. Lett. **66**, 2931 (1991).

[7] J. D. Reger and A. P. Young, *Monte Carlo study of a vortex glass model*, Journal of Physics A Mathematical General **26**, 1067 (1993).

[8] K. Hukushima and K. Nemoto, *Exchange Monte Carlo method and application to spin glass simulations*, J. Phys. Soc. Japan **65**, 1604 (1996).

[9] E. Marinari, *Optimized Monte Carlo methods*, in *Advances in Computer Simulation*, edited by J. Kertész and I. Kondor (Springer Verlag, Berlin, 1998), p. 50, (cond-mat/9612010).

[10] J. D. Reger, T. A. Tokuyasu, A. P. Young, and M. P. A. Fisher, *Vortex-glass transition in three dimensions*, Phys. Rev. B **44**, 7147 (1991).

[11] M. J. P. Gingras, *Numerical study of vortex-glass order in random-superconductor and related spin-glass models*, Phys. Rev. B **45**, 7547 (1992).

[12] J. M. Kosterlitz and M. V. Simkin, *Numerical Study of a Superconducting Glass Model*, Phys. Rev. Lett. **79**, 1098 (1997).

[13] J. Maucourt and D. R. Grempel, *Phase transitions in the two-dimensional XY model with random phases: A Monte Carlo study*, Phys. Rev. B **56**, 2572 (1997).

[14] N. Akino and J. M. Kosterlitz, *Domain wall renormalization group study of xy model with quenched random phase*

shifts, (cond-mat/0203299) (2002).

[15] M. Cieplak, J. R. Banavar, M. S. Li, and A. Khurana, *Frustration, scaling, and local gauge invariance*, Phys. Rev. B **45**, 786 (1992).

[16] M. A. Moore and S. Murphy, *Calculation of the exponent μ for the gauge glass model*, Phys. Rev. B **50**, 3450 (1994).

[17] J. M. Kosterlitz and N. Akino, *Numerical Study of Order in a Gauge Glass Model*, Phys. Rev. Lett. **81**, 4672 (1998).

[18] H. G. Katzgraber and A. P. Young, *Nature of the spin-glass state in the three-dimensional gauge glass*, Phys. Rev. B **64**, 104426 (2001), (cond-mat/0105077).

[19] K. Binder, *Critical properties from Monte Carlo coarse graining and renormalization*, Phys. Rev. Lett. **47**, 693 (1981).

[20] D. S. Fisher and D. A. Huse, *Absence of many states in realistic spin glasses*, J. Phys. A **20**, L1005 (1987).

[21] D. A. Huse and D. S. Fisher, *Pure states in spin glasses*, J. Phys. A **20**, L997 (1987).

[22] D. S. Fisher and D. A. Huse, *Equilibrium behavior of the spin-glass ordered phase*, Phys. Rev. B **38**, 386 (1988).

[23] D. S. Fisher, M. P. A. Fisher, and D. A. Huse, *Thermal fluctuations, quenched disorder, phase transitions, and transport in type-II superconductors*, Phys. Rev. B. **43**, 130 (1991).

[24] H. G. Katzgraber, M. Palassini, and A. P. Young, *Monte Carlo simulations of spin glasses at low temperatures*, Phys. Rev. B **63**, 184422 (2001), (cond-mat/0007113).