Chaos in a Two-Dimensional Ising Spin Glass

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We study chaos in a two dimensional Ising spin glass by finite temperature Monte Carlo simulations. We are able to detect chaos with respect to temperature changes as well as chaos with respect to changing the bonds, and find that the chaos exponents for these two cases are equal. Our value for the exponent appears to be consistent with that obtained in studies at zero temperature.

I. INTRODUCTION

A characteristic feature of spin glasses is that the relative orientations of the spins in the spin glass state are not uniquely determined by the model, but rather vary with external parameters such as the temperature or the magnetic field. At large separation, R, the correlation between the spins varies in a chaotic manner as a function of temperature, and, when the temperature is altered by an amount ΔT, will change substantially at distances R greater than $l_{\Delta T}$ where

$$l_{\Delta T} \sim (\Delta T)^{-1/\zeta}, \tag{1}$$

which defines the chaos exponent ζ. This temperature induced chaos has been quite difficult to see in Monte Carlo (MC) simulations[1,2] or mean field theory[3,4], and claims have been made that is absent or very small[1–3]. A larger chaotic effect has been observed when making a small change in the couplings[5,6].

Chaos with temperature has been shown to be a common feature of the two main models for the spin glasses phase: mean field theory[3,4] and the droplet theory[7–10]. The later is based upon real space renormalization group calculations which allow for quantitative results. In particular, they indicate that a temperature perturbation generates a disorder perturbation and thus these two perturbations should have the *same* chaos exponent[10].

Here we study chaos with both ΔT and a change in the couplings, ΔJ (defined precisely in Eq. (6) below), by Monte Carlo simulations for a two-dimensional Ising spin glass at finite temperature. Since $T_c = 0$ for this model, all our data is in the paramagnetic phase, where the correlation function tends to zero at large distances, so we are looking for chaos in the sign of this decaying function (such chaos has been shown in a one dimensional system[9]). We study distances which are smaller than the correlation length so the chaos we obtain is that corresponding to the critical point[10,11].

Our main results are as follows:

1. It *is* possible to see chaos with ΔT as well as chaos with ΔJ.
2. The chaos exponents for ΔT and ΔJ appear to be equal.
3. The chaos exponent found here at finite-T seems to be consistent with that obtained at $T = 0$[6,7],

The plan of the paper is as follows: In §II we define the model and various quantities of interest. §III discusses finite-size effects which will be very important for the analysis while §IV presents the numerical results that are then interpreted in §V.

II. THE MODEL

We consider the Edwards-Anderson Hamiltonian with Ising spins and nearest neighbor couplings,

$$\mathcal{H} = -\sum_{\langle i,j\rangle} J_{ij} S_i S_j \ , \tag{2}$$

where the $\{J_{ij}\}$ are drawn from a Gaussian distribution with zero mean and variance $[J_{ij}^2]_{av}$ equal to unity. We denote by $[\cdots]_{av}$ an average over the interactions. The spins lie on a square lattice of linear size L with periodic boundary conditions.

For each realization of the disorder we simulate several copies (or replicas) of the system. The basic quantity we calculate is the replica overlap

$$q_{ab} = \frac{1}{N}\sum_{i=1}^{N} S_i^{(a)} S_i^{(b)} \ , \tag{3}$$

where a and b denote replicas and $N = L^2$. When we investigate chaos with ΔT, some of the replicas will have identical bonds but slightly different temperatures and when we investigate chaos with ΔJ some of the replicas will have slightly different couplings but the same temperature.

Next we describe quantities that we calculated in the simulations. First of all, from replicas with the *same* temperatures and bonds, we compute the standard equilibrium quantities, g, the Binder ratio, and χ_{SG}, the spin glass susceptibility, defined by

$$g \equiv \frac{1}{2}\left[3 - \frac{\langle q^4\rangle}{\langle q^2\rangle^2}\right] \tag{4}$$

$$\chi_{SG} \equiv L^2 \langle q^2\rangle \ , \tag{5}$$

where $\langle\cdots\rangle$, denotes both the average over disorder and the statistical mechanics (Monte Carlo) average. During the simulation, the first t_0 sweeps are used for equilibration and the next t_0 sweeps are used for measurements.

We check that the system is in equilibrium by standard methods[12]. The equilibration time t_0 limits the maximum size and minimum temperature we can study. In our case, we can reach $T = 0.4$ for $L = 6$ and $T = 0.55$ for $L = 10$, which both require about $t_0 = 10^6$ MC steps.

Next we describe quantities that we calculate to determine the chaos. First, consider chaos with changing the bonds, keeping the temperature fixed. This is done by running one replica with a set of bonds $\{J_{ij}\}$ and another with bonds $\{J'_{ij}\}$, where[10]

$$J'_{ij} = \frac{J_{ij} + x_{ij}\Delta J}{\sqrt{1+\Delta J^2}}, \tag{6}$$

where x_{ij} is a Gaussian random variable with zero mean and unit variance. Note that the $\{J'_{ij}\}$, and the $\{J_{ij}\}$ have the *same* distribution. A convenient measure of how much the change in the bonds alters the spin orientations is the dimensionless "chaos parameter"[5],

$$r_{\Delta J} \equiv \frac{\langle q^2_{JJ'}\rangle}{\langle q^2_{JJ}\rangle}, \tag{7}$$

where the labels on the replicas refer to the bond distributions that are used.

When the temperature is changed we consider the overlap from replicas at temperatures *symmetrically* displaced about T as follows:

$$r_{\Delta T} \equiv \frac{\langle q^2_{T_+T_-}\rangle}{\sqrt{\langle q^2_{T_+T_+}\rangle\langle q^2_{T_-T_-}\rangle}}, \tag{8}$$

where the temperatures are $T_\pm = T \pm \Delta T/2$. We believe that this is the first time that chaos with temperature has been calculated in this way. Other attempts[1] to look for chaos with temperature used a quantity which has an asymmetry in temperature and will therefore involve bigger corrections to scaling when the ratio $\Delta T/T$ is not vanishingly small.

We average over from 80 to 400 realizations of the disorder. Error bars are determined by grouping the results for the different samples into bins and calculating the standard deviations among bins[13].

III. FINITE SIZE EFFECTS

To be in the scaling regime it is necessary to work at moderately low temperatures where finite size effects are important, and so finite-size scaling[14] techniques are needed.

Since $\xi \sim T^{-\nu}$, where ξ is the *bulk* correlation length, and since, at the $T = 0$ critical point, the ground state is unique, finite-size scaling predicts the following behavior for the Binder ratio and χ_{SG}:

$$g = \tilde{g}\left(L^{1/\nu}T\right) \tag{9}$$

$$\chi_{SG} = L^2\tilde{\chi}_{SG}\left(L^{1/\nu}T\right). \tag{10}$$

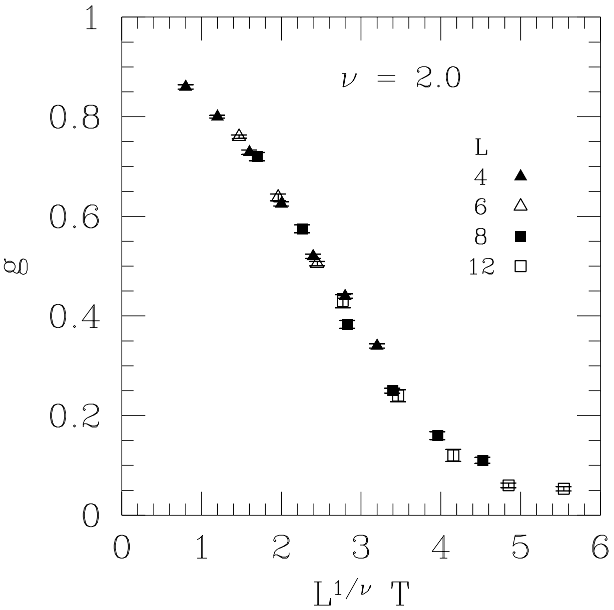

FIG. 1. A scaling plot of the Binder ratio according to Eq. (9) for different sizes and temperatures. The value of the correlation length exponent is $\nu = 2.0$.

We also need the finite size scaling ansatzes for the chaos parameters, $r_{\Delta J}$ and $r_{\Delta T}$:

$$r_{\Delta J} = \tilde{r}_{\Delta J}\left(L^{\zeta}\Delta J, L^{1/\nu}T\right) \tag{11}$$

$$r_{\Delta T} = \tilde{r}_{\Delta T}\left(L^{\zeta}\Delta T, L^{1/\nu}T\right), \tag{12}$$

where we have used Eq. (1). It is inconvenient to analyze a function of two variables, so, following a suggestion of D. Huse, we have taken data where the second argument $L^{\frac{1}{\nu}}T$, is roughly constant. We then try to collapse the data on to a single curve by plotting it against $L^{\zeta}\Delta J$ with a suitable choice of ζ.

IV. RESULTS

Scaling plots for the Binder ratio and spin glass susceptibility are shown in Figs. 1 and 2. In Fig. 1 the sizes L range from 4 to 12 and temperatures T from 0.4 to 1.6. The resulting correlation length exponent at the $T = 0$ transition is $\nu = 2.0 \pm 0.2$. For Fig. 2 the sizes extend from $L = 4$ to $L = 20$, at temperatures from 0.4 to 1.6, and the correlation length exponent is given by $\nu = 1.6 \pm 0.2$.

The value for ν obtained from g is in good agreement with finite temperature Monte Carlo simulations of Liang[15] and also agrees with the work of Kawashima et al.[16]. The estimate for ν obtained from χ_{SG} is somewhat smaller, presumably reflecting the systematic corrections to finite size scaling at the temperatures and sizes that we can study. Interestingly the same trend, namely a

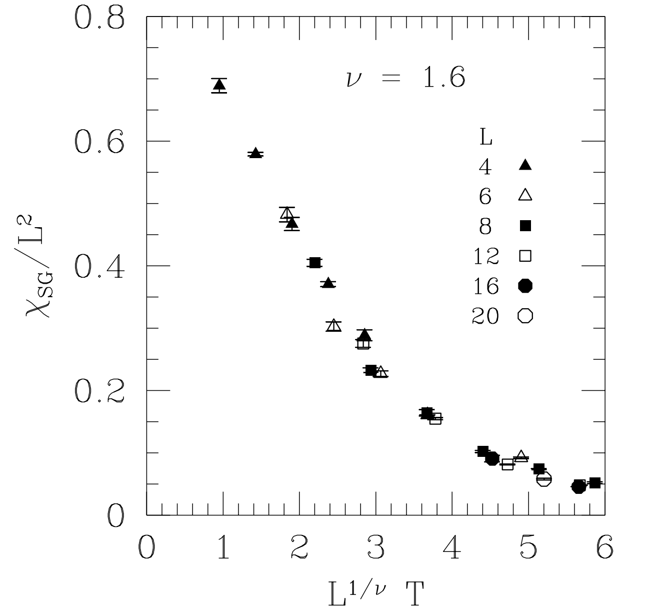

FIG. 2. A scaling plot of χ_{SG} according to Eq. (10) for different sizes and temperatures. The value of the correlation length exponent is $\nu = 1.6$.

larger value for ν obtained from g than from χ_{SG}, has been seen in other models[17].

Combining the two values for ν we estimate

$$\bar{\nu} \;=\; 1.8 \pm 0.4\;. \tag{13}$$

We also determined the spin-spin correlation function for temperatures between 0.8 and 1.2. By fitting this data to an exponential function of position, we estimate the correlation length, finding that it could be fitted to $\xi = AT^{-\nu}$ with $A = 4 \pm 0.5$ and $\nu = 1.8 \pm 0.2$, the latter being consistent with the estimates from the finite-size scaling analysis above.

A scaling plot for chaos with ΔJ, following Eq. (11) with $L^{1/\nu}T$ constant (and $\nu = 1.8$), is shown in Fig. 3, for sizes between 4 and 10 with $\zeta = 1.0$. The perturbation, ΔJ, lies in the range $0.05 - 0.3$. Trying different values of ζ we estimate

$$\zeta \;=\; 1.0 \pm 0.1 \quad (\text{chaos with } \Delta J)\,. \tag{14}$$

A scaling plot for chaos with ΔT, following Eq. (12) with $L^{1/\nu}T$ constant (and $\nu = 1.8$), is shown in Fig. 4, for $L \leq 10$, and $\zeta = 1.0$. The perturbation, ΔT, lies in the range $0.05 - 0.4$. Trying different values of ζ we estimate

$$\zeta \;=\; 1.0 \pm 0.2 \quad (\text{chaos with } \Delta T)\;. \tag{15}$$

Note that the exponents for chaos with ΔJ and ΔT, given in Eqs. (14) and (15), are equal within the uncertainties. We also see from Figs. 3 and 4 that the data for $r_{\Delta T}$ does not deviate very much from unity, as compared

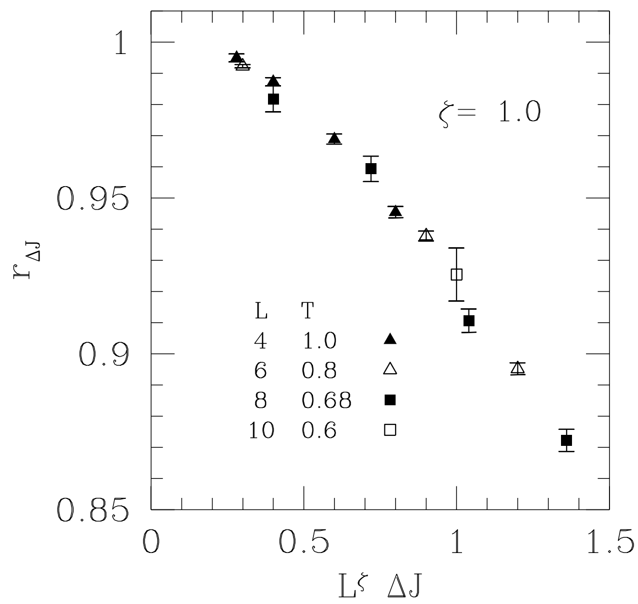

FIG. 3. A scaling plot of $r_{\Delta J}$ according to Eq. (11) with $L^{1/\nu}T$ constant and $\nu = 1.8$, the average of our estimates from g and χ_{SG}. The value of the chaos exponent is $\zeta = 1.0$.

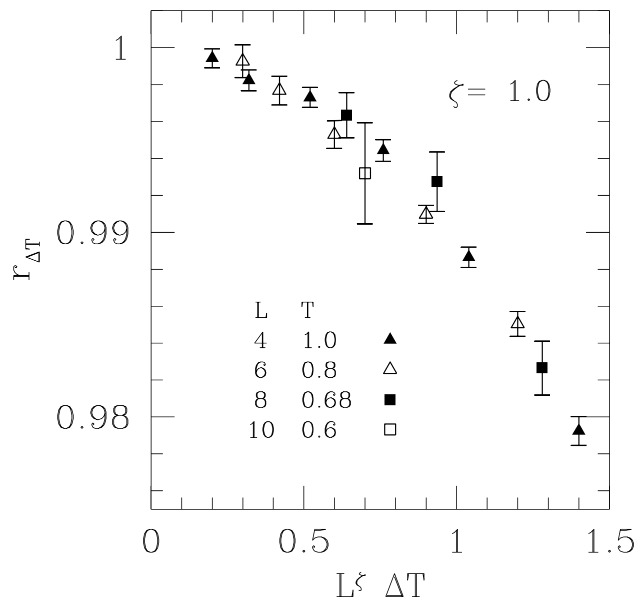

FIG. 4. A scaling plot of $r_{\Delta T}$ according to Eq. (12) with $L^{1/\nu}T$ constant and $\nu = 1.8$, the average of our estimates from g and χ_{SG}. The value of the chaos exponent is $\zeta = 1.0$.

with the data for $r_{\Delta J}$. This indicates that the *amplitude* of chaos with ΔT is smaller than that with ΔJ, even though the exponents are equal.

V. DISCUSSION

Studying the two-dimensional Ising spin glass by finite temperature Monte Carlo simulations, we are able to detect chaos with respect to both ΔJ and ΔT and show that the chaos exponents are equal, as expected[10].

We should point out that in order to define chaos with ΔT in the critical region it is necessary that $\nu > 1/\zeta$. To see this note that we need $l_{\Delta T}$ in Eq. (1) to be less that the correlation length, $\xi \sim T^{-\nu}$, and also $\Delta T \ll T$ to be in the scaling region for chaos. In our case, this inequality is satisfied (note that chaos in the critical region is also present when $T_c > 0$[10,18]). Chaos with ΔJ, on the other hand, can be defined irrespective of the relative values of ν and $1/\zeta$.

Our estimates of the exponent, given in Eqs. (14) and (15), are consistent with the value $\zeta = 0.95 \pm 0.05$[6] found from exact ground state determinations. A similar value was also found earlier by Bray and Moore[7].

Finally we note that our value for ν agrees with work of Liang[15] who gets $\nu \approx 2$ from Monte Carlo simulations. However a much larger value is inferred at $T = 0$, from domain wall renormalization group calculations[19], i.e. $\nu = 4.2 \pm 0.5$, and from exact ground state calculation[6], i.e. $\nu = 3.559 \pm 0.025$. These discrepancies suggest a violation of the scaling picture of the spin glass transition. Kawashima et al.[16] also find a discrepancies in the scaling theory. If there are violations of the scaling picture in two dimensions it would be very valuable to understand them since similar violations may also occur in higher dimensions with a finite T_c, and also perhaps help resolve disagreements between the droplet and mean field pictures.

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