PHYSICS 219

Monte Carlo Project

To be handed in on or before the final exam, 5 pm. June 12.

Perform an importance sampling Monte Carlo simulation for the two-dimensional Ising model. Consider $L \times L$ lattices with L increasing in a geometrical manner, e.g. 4, 8, 16, 32 and possibly larger. Use periodic boundary conditions. Perform simulations at different temperatures in the vicinity of T_c. The relaxation time at the critical point increases like L^z, where z, known as a "dynamical exponent", is roughly 2. You will therefore need to perform more sweeps to equilibrate and do the averaging at larger L.

1. Compute the values of $\langle m^2 \rangle$ where

$$m = \frac{1}{L^2} \sum_i S_i$$

   is the magnetization per site. Present the results graphically.
   You should find that this quantity increases in the vicinity of the transition temperature, with the increase being more sudden for the larger sizes.

2. According to the theory of finite-size scaling, the behavior of should have the form

$$\langle m^2 \rangle = L^{-2\beta/\nu} \widetilde{X} \left[L^{1/\nu} (T - T_c) \right] ,$$

   where $\widetilde{X}$ is called a "scaling function", β is the order parameter exponent, and ν is the exponent describing the diveregence of the correlation length. You are given that for the $d = 2$ Ising model $\nu = 1$. The values of T_c and β were discussed in class. Show that your data fits this form pretty well by plotting $L^{2\beta/\nu} \langle m^2 \rangle$ against $L^{1/\nu}(T - T_c)$.

3. Why do you think I asked you compute $\langle m^2 \rangle$ rather than $\langle m \rangle$?

Note: A professional calculation would include (a) error bars on the data points, and (b) tests for equilibration. I'm not *requiring* these here but, of course, you can incorporate them if you wish. To get error bars you could repeat the calculation several times (most convenient to do this within a single run). To test for equilibration, the most common approach is to increase the length of simulation by some factor (e.g. 2 or 10) and verify that the answers do not change within the error bars.