PHYSICS 116C
Homework 6
Due in class, Thursday November 7

- The **midterm** will be in class on **Tuesday November 5**. It will cover all the material up to and including separation of variables in cartesian coordinates.
- Exams are **closed book** but you may bring one sheet of notes that you have prepared yourself.
- **No calculators** are allowed in exams.
- **In exams (and homework) it is essential that you EXPLAIN your work.**
- This problem set is shorter than usual to give you time to **fully prepare** for the midterm.
- To prepare for the midterm, it us useful to **go over the homework problems**, for which typed up solutions are available on the class web site

  `http://young.physics.ucsc.edu/116C`.

  Also solve similar problems from the book.
- Read both sides of this sheet.

1. Consider separation of variables of Laplace's equation in circular polars.

   (a) Show that the angular equation is
   $$\frac{d^2\Theta}{d\theta^2} + n^2\Theta = 0 \quad (n = 0, 1, 2, \cdots)\,.$$
   What is the solution of this equation (easy) and why must n be an integer?

   (b) Show that the radial equation is
   $$r\frac{d}{dr}\left(r\frac{dR}{dr}\right) = n^2 R \quad (n = 0, 1, 2, \cdots)\,.$$
   Look for solutions of the form $R = r^\lambda$ (i.e. just a single power of r) for a suitable choice of λ. *Note:* You will need to treat the $n = 0$ case separately.

   (c) Write down the general solution of Laplaces's equation in circular polars.

2. Consider the electrostatic potential inside a semi-infinite cylinder of radius a and occupying the range $0 \le z < \infty$. The potential is zero on the round vertical surface at $r = a$, and given by $r \sin\theta$ on the bottom face at $z = 0$. Find the potential inside the cylinder, expressing your answer as a series.
   Note: The series will involve zeroes of a Bessel function which you should not evaluate numerically.
   Hints:

   (a) The solution will contain $\sin\theta$ and hence it must also involve J_1.

   (b) You are given the following results:
   $$\begin{aligned}\int_0^1 x\, J_m(c_{n,m}x) J_m(c_{p,m}x)\, dx &= \delta_{np}\,\frac{1}{2}\, J_{m+1}^2(c_{n,m}) \\ \frac{d}{dx}\left[x^n J_n(x)\right] &= x^n J_{n-1}(x)\end{aligned}$$
   Here $c_{n,m}$ is the n-th zero of $J_m(x)$, *i.e.* $J_m(c_{n,m}) = 0$.

3. Consider a cylinder of radius $r = a$ and height $z = L$. Its top and bottom faces are maintained at $T = 0$ and its curved surface is maintained at $T = T_0 z(L - z)$. Consider the steady state temperature distribution (which satisfies Laplace's equation).

   (a) By separating out the part of the solution which depends on z show that the equation determining the solution in the r–θ plane is

   $$\left(\nabla^2 - k^2\right) u(r, \theta) = 0\,,$$

   (where ∇^2 refers to the Laplacian in circular polars) which differs from the Helmholtz equation in the minus sign in front of k^2.

   (b) Noting the solution is independent of θ show that the radial part of the solution can be written as

   $$R_n(r) = c_1 I_0\left(\frac{n\pi r}{L}\right) + c_2 K_0\left(\frac{n\pi r}{L}\right)\,,$$

   where I_0 and K_0 are modified Bessel functions of the first and second kind. (You may need to look up their properties.)

   (c) Explain why we must have $c_2 = 0$.

   (d) Hence determine the solution in the form of a series.