PHYSICS 116C
Homework 5
Due in class, Thursday October 31

Midterm Reminder: The midterm will be in class on Tuesday November 5. The exam will be closed book, but you may bring one one sheet of handwritten notes on which you can write anything you like. No electronic devices will be allowed.
It will cover material through Boas Sec. 13.4 (i.e. partial differential equations in Cartesian coordinates.)

1. (a) Determine the steady-state temperature distribution in a square plate of size L if one side is held at temperature T_0 and the other three sides are held at $T = 0$.
   Note: Your answer will be expressed as an infinite series.

   (b) Keeping just a small number of terms in the series, estimate numerically the temperature at the center.

   (c) What is the exact value of the temperature at the center?
   Hint: Consider *superposition* of the four distinct solutions in which one of the four sides is at $T = T_0$ and the others are at $T = 0$.

2. Consider a bar of length L, whose temperature is fixed at both ends to be $T = 0$. The initial temperature dependence in the bar is $T(x, 0) = T_0\, x(L - x)$. Use the diffusion equation

$$\frac{\partial T}{\partial t} = D\, \frac{\partial^2 T}{\partial x^2}$$

   to determine the temperature at later times.

3. A slab of length L is initially in steady state with one end at temperature T_1 and the other end at T_2. At time $t = 0$, the temperatures are suddenly interchanged. Find $T(x, t)$ for $t > 0$.
   Note: You will need to include the solution with zero separation constant.

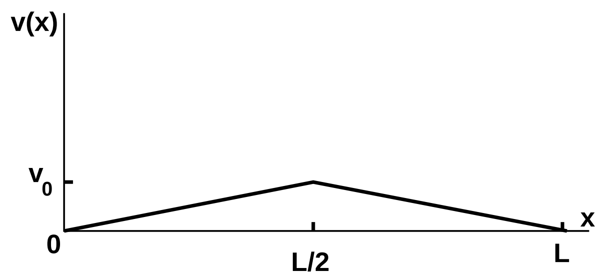
4. Solve the wave equation for a string of length L clamped at the ends given that the initial conditions are $y(x, 0) = y_0 \sin(3\pi x/L), \partial_t y(x, t)|_{t=0} = 0$, where $y(x, t)$ is the displacement.

5. (a) Show that

$$\sin 3x = 3 \sin x - 4 \sin^3 x\,.$$

   (b) Solve the wave equation for a string of length L clamped at the ends given that the initial conditions are $y(x, 0) = y_0 \sin^3(\pi x/L), \partial_t y(x, t)|_{t=0} = 0$.

6. A string of length L is initially stretched straight. At time $t = 0$, it is given an initial *velocity* $v(x) = \partial_t y(x, t)|_{t=0}$ as shown.



Find the displacement $y(x,t)$ at later times.

7. (a) We showed in class that the general solution of the one-dimensional wave equation is

$$u(x,t) = f(x - vt) + g(x + vt)\,,$$

where f and g are arbitrary functions. Show that if we specify the initial conditions

$$u(x,0) = \phi(x)\,, \qquad \partial_t u(x,t)|_{t=0} = \psi(x)\,,$$

then the solution is

$$u(x,t) = \tfrac{1}{2}\left[\phi(x - vt) + \phi(x + vt)\right] + \frac{1}{2v}\int_{x-vt}^{x+vt} \psi(z)\,dz\,.$$

(This is known as d'Alembert's solution to the one dimensional wave equation.)
Hint: Look in the book.

(b) Use d'Alembert's solution to find the solution to the wave equation that satisfies the initial conditions $u(x,0) = \sin(\pi x/L), \partial_t u(x,t)|_{t=0} = 0$ for $-\infty < x < \infty$. Express your answer as a standing wave.