PHYSICS 115

FINAL EXAM, 2014

Due in my mailbox in ISB 232 by 5 pm. on Tuesday, June 10.

This must be your own work. **No collaboration is allowed.** You may use your notes, books, and the web.

For the C/C^{++}/Fortran/Java questions you must write a program in C, C^{++}, Java or Fortran, but not use Mathematica. (Of course, nothing stops you from *checking* your answer with Mathematica.) You must provide a listing of the program, the output, and any relevant explanation. Group all parts of a question together.
For the Mathematica questions you must provide a listing of the commands you used, the output, and any relevant explanation. Put all your answers to the Mathematica part in one notebook.

C/C^{++}/Fortran/Java part

1. Consider the following data:

```
#      x         y      error in y
    1.1075    5.1807    0.0813
    2.0198    7.0209    0.0811
    2.8948    8.7185    0.0487
    3.8009   10.5185    0.0736
    4.8840   12.7217    0.0621
    6.1507   15.3488    0.1496
    7.0411   17.1824    0.0853
    8.0592   19.1125    0.1110
    8.8211   20.7522    0.1253
   10.0766   23.5099    0.1433
```

(Download this data from `http://young.physics.ucsc.edu/115/homework/qu1fitdata`.)

(a) Determine the best straight-line fit to this data using the least-squares method. You need to determine the error bars on the slope and intercept as well as the best-fit values of these parameters.

(b) Determine the number of degrees of freedom, the value of χ^2, and hence the value of χ^2 per degree of freedom.

2. Evaluate

$$I = \int_0^2 \frac{1}{1+x^3}\, dx$$

numerically to 6 decimal places.
Note: You must explain how you estimated that the desired accuracy had been obtained.

3. Find the positive root of the equation

$$x = \sin 3x,$$

to three decimal places.
Note: You must explain how you estimated that the desired accuracy had been obtained.

4. Consider the differential equation

$$\frac{dy}{dx} = \sqrt{y + x^2}.$$

Find the value of $y(2)$ correct to 4 decimal places given that $y(0) = 1$.
Note: You must explain how you estimated that the desired accuracy had been obtained.

5. Compute the following integral by Monte Carlo methods:

$$I = \int_0^2 dx_1 \int_0^2 dx_2 \cdots \int_0^2 dx_8 \, \frac{1}{1 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8} \, .$$

You must indicate how many points you generated and give an estimate for the error bar.

Mathematica part

6. (a) Find numerically all the solutions of

$$x^8 - 4x^3 = 3 \, .$$

(b) Determine how many integers between 10^6 and 1.5×10^6 are perfect squares, (e.g. like 64 which is equal to 8^2.)

(c) Find analytically (using Mathematica) the value of the following integral

$$\int_0^\infty \frac{1}{(1 + x^n)^m} \, dx \, ,$$

assuming $n > 0, m > 0, nm > 1$. Find a simpler expression, not involving Gamma functions, for the case of $m = 1$.

7. In appropriate units, mean field theory predicts that the magnetization m of an Ising magnet *in a magnetic field* h is given by

$$m = \tanh\left(\frac{Jm + h}{T} \right) \, .$$

We will take $h = 0.001$ and you are given that the physical solution is the one with $m > 0$. Set the interaction J to be unity.

(a) What is m at $T = 1$?

(b) Using a one-line command, plot m versus T, with T ranging from a value close to zero up to $T = 1.3$.

8. Consider the logistic map

$$x_{n+1} = 4\lambda x_n (1 - x_n),$$

for $\lambda = 9/10$.

(a) Compute the Lyapunov exponent and decide whether this value of λ is in the chaotic region or not.

(b) Starting with $x_0 = 3/10$, compute x_1, x_2 and x_{5000}.
You should give your results to, say, at least 5 decimal places, and for the last value, explain what you did to be confident of the result.

9. Consider a particle of unit mass in the double well potential

$$V(x) = \begin{cases} \dfrac{V_0}{2} \left[1 - \cos\left(\dfrac{4\pi x}{L} \right) \right] & \text{for } |x| \le L/2 \\ 0 & \text{for } |x| > L/2 \end{cases}$$

with $L = 2$. Work in units with $\hbar = m = 1$ and take $V_0 = -50$.

(a) Plot $V(x)$.

(b) Determine the three lowest energy levels and plot the corresponding normalized wave-functions.