PHYSICS 110A

Midterm 2009

In class, Thursday, February 12, 2:00–3:45 pm.

The exam is closed book, but you may bring in one set of notes that you have prepared yourself (no photocopies). No electronic devices may be used during the exam.

1. [**30 points**] **Vector Calculus**

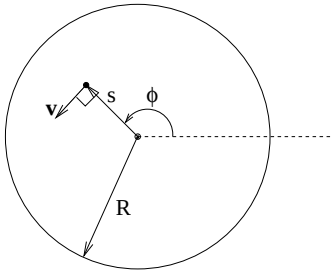
   (a) Calculate (i) $\boldsymbol{\nabla} r$, (ii) $\boldsymbol{\nabla} \cdot \mathbf{r}$, (iii) $\boldsymbol{\nabla} \times \mathbf{r}$.

   (b) For $\mathbf{A} = 3\hat{\mathbf{z}}, \mathbf{B} = (2x^2 + z)\,\hat{\mathbf{x}} + xz\,\hat{\mathbf{y}} + 3x\,\hat{\mathbf{z}}$ compute (i) $\mathbf{A}\,(\boldsymbol{\nabla} \cdot \mathbf{B})$, (ii) $(\mathbf{A} \cdot \boldsymbol{\nabla})\,\mathbf{B}$.

   (c) What is $\nabla^2\,(x^2 + 2xy + z + 4)$?

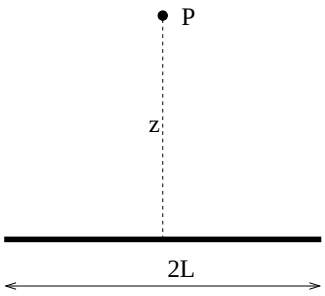
   (d) For $\mathbf{C}(\mathbf{r}) = k\mathbf{r}$ verify Gauss's law, $\oint_S \mathbf{C} \cdot d\mathbf{a} = \int_V \boldsymbol{\nabla} \cdot \mathbf{C}\, d\tau$, over a sphere of radius R centered at the origin.

   (e) Consider the function $\mathbf{v}(\mathbf{r}) = k\,s\,\hat{\boldsymbol{\phi}}$ in *cylindrical* polar coordinates. Verify Stokes's theorem, $\oint_P \mathbf{v} \cdot d\boldsymbol{\ell} = \int_S (\boldsymbol{\nabla} \times \mathbf{v}) \cdot d\mathbf{a}$ where P is the circumference and S the area of a circle of radius R centered at the origin (see figure).

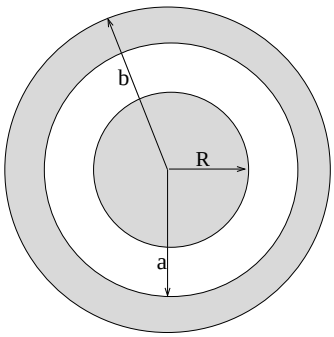


2. [**15 points**]

   Compute the electrostatic potential a distance z above the center of a line charge of length $2L$ carrying charge λ per unit length (see figure).
   Note: You are given that $\int \frac{dx}{\sqrt{a^2+x^2}} = \ln(x + \sqrt{x^2 + a^2})$.



3. [**20 points**]



   A conducting sphere of radius R carrying a charge q is surrounded by a thick concentric conducting shell with inner radius a and outer radius b (see figure). The shell contains no net charge.

   (a) Find the surface charge density on the surfaces at R, a and b.

   (b) Find the field (i) outside the shell and (ii) between the shell and the sphere.

   (c) Find the potential at the origin.
   Note: As a point of reference take the potential to be zero at infinity.

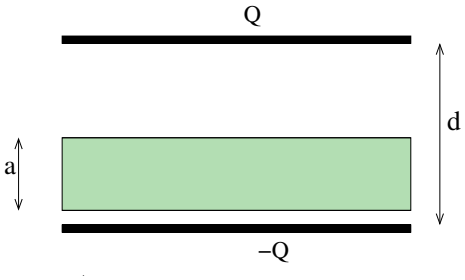
4. [**15 points**]
In class we showed that a conducting sphere of radius R in a uniform external field carries a surface charge $\sigma = k\cos\theta$, where k is a constant.

   (a) What is the total charge of this distribution?

   (b) Compute the dipole moment of this distribution.

5. [**20 points**]

Consider a parallel plate capacitor which has a distance d between the plates. *Part* of the space between the plates is filled with a dielectric of width a ($< d$) and dielectric constant ϵ_r. The plates have an area A and have (free) charge Q on the top plate and $-Q$ on the bottom plate.



   (a) Determine the "electric displacement **D**" and the electric field **E** in the region between the plates (both in the dielectric and in the empty space).

   (b) Determine the potential difference V between the plates.

   (c) Determine the capacitance C of the plates.
   Recall: $Q = CV$.