PHYSICS 110A

Homework 7

Due in class, Tuesday, February 24.

1. Two infinite straight line charges, λ per unit length, are distance d apart moving at constant speed v (see Griffiths Fig. 5.26). How great would v have to be for the magnetic attraction to balance the electric repulsion?

2. A steady current I flows down a long cylindrical wire of radius a. Find the magnetic field, inside and outside the wire, if

   (a) The current is uniformly distributed over the *surface* of the wire.

   (b) The current is uniformly distributed over the *bulk* of the wire (i.e. the current density $\mathbf{J}$ is the same everywhere in the wire).

3. Two long coaxial solenoids each carry current I, but in opposite directions, as shown in Griffiths Fig. 5.42. The inner solenoid, of radius a has n_1 turns per unit length, and the outer one, of radius b has n_2. Find $\mathbf{B}$ in each of the three regions: (i) inside the inner solenoid, (ii) between the solenoids, and (iii) outside both of them.

4. In calculating the current enclosed by an Ampèrian loop, one must evaluate an integral of the form
$$I_{\text{enc}} = \int_{\mathcal{S}} \mathbf{J} \cdot d\mathbf{a}.$$
However, there in infinitely many surfaces that share the same boundary line. Which one are we supposed to use?
Note: If it doesn't matter, i.e. if they all give the same answer, the you must explain *why*.

5. (a) Find the density ρ of mobile charges in a piece of copper, assuming each atom contributes one free electron.
   Note: Some useful constants are: density of copper is 9.0 gm/cm^3, atomic mass of copper = 64 gm/mole. Other constants you should look up.

   (b) Calculate the average electron velocity in a copper wire 1 mm in diameter, carrying a current of 1 A.
   Note how amazingly small it is.

6. (a) If $\mathbf{B}$ is uniform show that $\mathbf{A}(\mathbf{r}) = -\frac{1}{2}(\mathbf{r} \times \mathbf{B})$ works, i.e. $\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}$ with $\boldsymbol{\nabla} \cdot \mathbf{A} = 0$.
   Note: It is simplest is to take $\mathbf{B}$ to be in a specified direction such as $\hat{\mathbf{z}}$.

   (b) Is this result unique or are there other functions with same divergence and curl?
   Note: It is again useful to take $\mathbf{B}$ to be in the z-direction, for which one has $\mathbf{A} = \frac{1}{2} B(-y, x, 0)$ in part (a). What about, for example $\mathbf{A} = B(0, x, 0)$ or $\mathbf{A} = B(-y, 0, 0)$?

7. The electrostatic potential of a line charge is computed from
$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \, dl'. \tag{1}$$

The vector potential of current along a wire is computed from a very similar expression

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{I}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}\, dl'. \tag{2}$$

We showed in Hw 3, Qu. 2, that for an infinite uniform line charge λ

$$V(s) = -\frac{\lambda}{2\pi\epsilon_0} \log\left(\frac{s}{a}\right),$$

where s is the distance to the wire. By using the analogy between Eqs. (1) and (2) write down the vector potential at a point a distance s from an infinite wire carrying a current I. Check that this leads to the expression for $\mathbf{B}$ obtained from Ampère's law that was discussed in class.

8. A phonograph record of radius R carrying a surface charge density σ, is rotating at a constant angular velocity ω. Find its magnetic dipole moment.

9. Magnetostatics treats the "source current" (the one that sets up the field) and the "recipient current" (the on that experiences the force) asymmetrically, so it is by no means obvious that the magnetic force between two current loops is consistent with Newton's third law. Show that the force on loop 2 due to loop 1 (Griffiths Fig. 5.62) can be written

$$\mathbf{F}_2 = -\frac{\mu_0}{4\pi} I_1 I_2 \oint \oint \frac{\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}\, d\mathbf{l}_1 \cdot d\mathbf{l}_2,$$

where $\boldsymbol{\mathscr{r}} = \mathbf{r}_2 - \mathbf{r}_1$. In this form it is clear that $\mathbf{F}_2 = -\mathbf{F}_1$ since $\hat{\boldsymbol{\mathscr{r}}}$ changes sign when the roles of 1 and 2 are interchanged.
Note: If you get an extra term it will be helpful to note that $\frac{\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2} = -\boldsymbol{\nabla}_2\left(\frac{1}{\mathscr{r}}\right)$.