# PHYSICS 110A

## Final Examination, 2009

Thursday, March 19, 7:30–10:30 pm.

The exam is closed book, but you may bring in one set of notes that you have prepared yourself (no photocopies). No electronic devices may be used during the exam.

$\boxed{\textbf{You must explain your work.}}$

1. [**20 points**]

   (a) What physical situation gives rise to the electric field

   $$\mathbf{E}(\mathbf{r}) = \frac{\hat{\mathbf{r}}}{r^2} \ ?$$

   State (without proof) the divergence of $\mathbf{E}$.

   (b) The electric potential of a charge configuration is given by the expression

   $$V(r) = A\,\frac{e^{-r}}{r},$$

   where $A$ is a constant. Find

   i. the electric field $\mathbf{E}(\mathbf{r})$,
   ii. the charge density, $\rho(r)$ and
   iii. the total charge.

2. [**20 points**]
   The potential on the surface of a charged spherical shell of radius $R$ is

   $$V_0(\theta) = k\,\cos 2\theta \qquad \left[= k\left(2\cos^2\theta - 1\right)\right],$$

   where $k$ is a constant. Find the potential inside and outside the sphere, as well as the charge density of the surface of the sphere.
   *Note:*

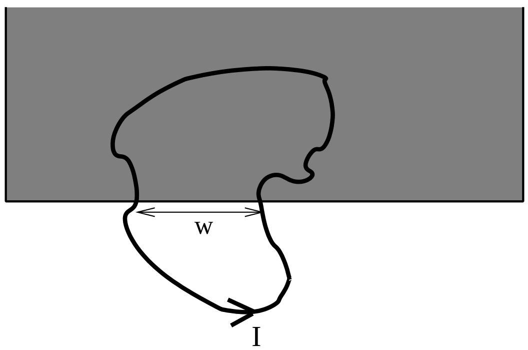
   - There's no charge inside or outside the spherical shell; only on its surface.
   - You are *given* that the solution of Laplace's equation in spherical polars with azimuthal symmetry is

     $$V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l\,r^l + \frac{B_l}{r^{l+1}}\right)\,P_l(\cos\theta),$$

     where $P_l(\cos\theta)$ is a Legendre polynomial.
   - You are also given that the first three Legendre polynomials are

     $$P_0(x) = 1, \quad P_1(x) = x, \quad P_2(x) = \tfrac{1}{2}\left(3\cos^2\theta - 1\right).$$

3. [**15 points**]




A plane wire loop of irregular shape is situated so that part of it is in a uniform magnetic field **B**. In the figure, the field is in the shaded region and is *into* the plane. The loop carries a current $I$.

(a) Show that the net magnetic force on the loop is $F = I\,B\,w$, where $w$ is the chord subtended as shown.

(b) Generalize this to the case where the region occupied by the field is also irregular in shape (but assume that the loop only enters and leaves the field once).
*Note:* Your answer should include a sketch.

(c) What is the direction of the force?

4. [**15 points**]
A current $I$ flows down a long straight wire of radius $a$. The wire is made of linear material (copper, say, or aluminum) with susceptibility $\chi_m$, and the current is distributed uniformly.

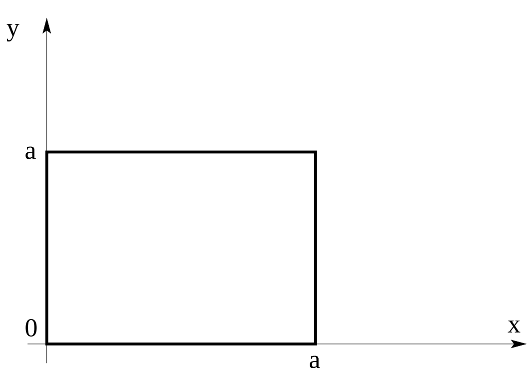
(a) By using Ampère's law for $\mathbf{H}$, compute $\mathbf{H}$ in the wire a distance $s$ ($< a$) from the axis.

(b) From part (a) determine $\mathbf{B}$ in the wire.

(c) The bound current is related to the magnetization by $\mathbf{J}_b = \boldsymbol{\nabla} \times \mathbf{M}$ and so, in a linear material as we have here, there is a simple relationship between $\mathbf{J}_b$ and $\mathbf{J}_f$, which you should find and use to compute the bound current $\mathbf{J}_b$.

(d) Hence, using Ampère's law for $\mathbf{B}$, determine $\mathbf{B}$ in the wire at a distance $s$ ($< a$) from the axis.

5. [**10 points**]




A square loop of conducting wire of resistance $R$ with sides of length $a$ lies in the first quadrant of the $xy$ plane with one corner at the origin as shown. There is a non-uniform time-dependent magnetic field

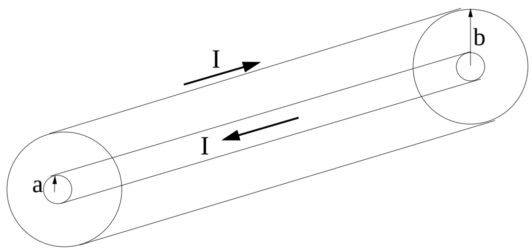
$$\mathbf{B}(y,t) = \begin{cases} 0, & (t < 0), \\ k\,y^2\,t\,(10-t)\,\hat{\mathbf{z}}, & (0 < t < 10), \\ 0, & (t > 10), \end{cases}$$

where $k$ is a constant.

(a) Find the current induced in the loop as a function of time.

(b) Find the total charge transported past a point on wire.

6. [**20 points**]




Consider a coaxial cable with an inner wire of radius $a$ and an outer cylindrical shell of radius $b$ (see figure). A current I flows along the inner wire and returns along the outer shell.

(a) Compute the magnetic field in between the wire and the shell at a distance $s$ from the axis of the cable, i.e. $a < s < b$.

(b) Hence find the magnetic inductance per unit length $\mathcal{L}$ of the cable.

(c) Now imagine that, instead of current, there is a *charge* $Q$ per unit length on the inner wire, and $-Q$ per unit length on the outer shell. Compute the potential difference between the inner wire and the outer shell, and hence determine the capacitance per unit length $\mathcal{C}$.

(d) What is the product $\mathcal{LC}$?
*Note:* the answer to 6d does not depend on the geometry of the cable. In fact one can show that this product is related to the speed with which a signal propagates down the cable by $v = 1/\sqrt{\mathcal{LC}}$.